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Drag Coefficient of a Spherical Particle Attached on the Flat Surface

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ABSTRACT

The previous analytical solution for the drag coefficient (C_d) for a spherical particle attached on the flat surface, which was derived by O'Neill (1968), is only valid in the creeping flow conditions. It is important to extend O'Neill's formula to cover a wide range of particle Reynolds number (Re_p). In this study, the drag coefficient was calculated numerically to cover Re_p from 0.1 to 250. For a particle suspended in the air, an empirical drag coefficient exists, which is defined as $C_d = f \times 24/Re_p$, where f is a correction factor depending on Re_p. The applicability of the correction factor f for O'Neill's analytical equation for the spherical particle attached on the flat surface for Re_p = 0.1 to 250 was examined in this study.

Keywords: Drag coefficient; Boundary layer; Numerical simulation; Reynolds number.

INTRODUCTION

Particle re-entrainment or detachment from solid surfaces is important in various engineering and environmental problems, such as surface cleaning, fugitive dust generation from unpaved surfaces and powder dispersion. Extensive literature has been published related to particle detachment from surfaces (Sehmel, 1980; Nicholson, 1988; Tsai *et al*., 1991a, b; Ziskind *et al*. 1995; Chiou and Tsai, 2001; Tsai and Chang, 2002; Tsai *et al*., 2003; Ziskind, 2006; Ibrahim *et al*., 2008; Gradon, 2009). The formation of a visible dust devil vortex depends on the presence of dust particles and the surface friction (Gu *et al*., 2010). There are three re-entrainment modes, namely direct lift-off, sliding and rolling. Among them rolling provides the least resistance for incipient detachment as compared to direct lift-off and rolling (Ibrahim *et al*., 2008). To determine if the particle detachment from the surfaces occurs, it is essential to calculate particle drag force accurately.

The previous analytical solution for the drag coefficient (C_d) for a spherical particle attached on the flat surface, which is only valid in the creeping flow conditions, was derived by O'Neill (1968) as

$$C_d = 1.7009 \frac{24}{Re_p} \quad (1)$$

where Re_p is the particle Reynolds number defined as

$$Re_p = \frac{\rho U_c D_p}{\mu} \quad (2)$$

where ρ, U_c, D_p, and μ are the density of air (kg/m^3), the velocity at the center of the particle (m/s), the particle diameter (m) and the air viscosity (kg/m-s), respectively. For $0.1 \le Re_p \le 250$, Sweeney and Finlay (2007) developed an empirical drag coefficient at the plate Reynolds number (Re_x) of 32,400 based on the numerical simulation as

$$C_d = \frac{40.812}{Re_p} \Bigg/ \left[1 - \frac{0.2817}{Re_p^{0.0826}} \operatorname{arcsinh}(0.238 Re_p)\right] \quad (3)$$

The plate Reynolds number is defined as

$$Re_x = \frac{\rho U_\infty x}{\mu} \quad (4)$$

where U_∞ is the free-stream velocity (m/s), x is the distance from the leading edge (m). Note that the inverse hyperbolic sine term in Eq. (3) was mistaken for inverse sine in Sweeney and Finlay (2007) (Martinez *et al*. 2009).

Ibrahim *et al*. (2008) used the following formula to calculate the drag force for the particle attached on the surface

$$C_d = 1.7009 \frac{24}{Re_p}[1 + 3Re_p/16 + 0.0079 Re_p^2 + 9 Re_p^2 \ln(Re_p)/160], \quad Re_p < 8 \quad (5)$$

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where the third term on the right hand side is the correction factor obtained from Ockendon and Evans (1972).

Up to now, very few studies on the drag coefficient of a spherical particle attached on the flat surface in a wide range of particle Reynolds number are available. It is desirable to have an accurate formula to predict the drag coefficient in non-creeping flow conditions. The objective of this study is to determine such drag coefficient numerically for $0.1 \le Re_p \le 250$.

For a particle suspended in the air, an empirical drag coefficient exists, which is defined as (Willeke and Baron, 1993)

$$C_d = f\,\frac{24}{Re_p} \tag{6}$$

where f is the correction factor expressed as

$$f = \begin{cases} 1 + 0.0916 Re_p & \text{for } 0.1 \le Re_p < 5 \\ 1 + 0.158 Re_p^{2/3} & \text{for } 5 \le Re_p < 1000 \end{cases} \tag{7}$$

This correction factor f was first checked by the present numerical simulation for Re_p from 10 to 200. The simulated drag coefficient for the particle attached on the flat surface was then compared with the empirical drag coefficient, Eq. (3). The correction factor f in Eq. (7) was then proposed to correct for O'Neill's analytical equation to obtain the drag coefficient for the particle attached on the flat surface as

$$C_d = 1.7009\, f\,\frac{24}{Re_p} \tag{8}$$

The accuracy of this equation was examined by the present numerical simulation in the range of Re_p from 0.1 to 250. The effect of plate Reynolds number, Re_x, on C_d was also studied.

NUMERICAL METHOD

In order to obtain the drag force acting on the spherical particle attached on the flat surface, a 3-D numerical simulation was conducted in this study. The computational domain is a rectangular box as shown in Fig. 1. A uniform free stream velocity u was set at the inlet boundary and the spherical particle of 100 µm in diameter was situated at the bottom at a distance x downstream of the inlet boundary, in which u = 5 to 144 m/s and x = 55 mm for high Re_x range of 18,310 to 527,000, and u = 0.2 to 74 m/s and x = 6.5 mm for low Re_x range of 87 to 32,000. Non-slip boundary condition was used on the particle and the bottom surfaces, while the prescribed pressure boundary of 101,325 Pa (1 atm) was applied on the remaining surfaces.

The governing equations are Navier-Stokes and continuity equations. Steady-state laminar incompressible flow was assumed. The Navier-Stokes and continuity equations were solved by using the STAR-CD 3.22 code (CD-adapco Japan Co., LTD) which is based on the finite volume discretization method. The pressure-velocity linkage was solved by the SIMPLE (semi-implicit method for pressure linked equation) algorithm (Patankar, 1980). The QUICK (quadratic upstream interpolation of convective kinematics) scheme was used to discretise the convection terms of Navier-Stokes equation. Hexahedral cells, which allows for finer grid spacing near the wall, were generated by an automatic mesh generation tool, TrueGrid (version 2.1.0, XYZ Scientific Applications, Inc.). The total number of grids used is 425,000 in the calculation domain. It was found that increasing the number of grids to 770,000 only resulted in a less than 1.2% difference in the C_d when the particle Reynolds number was set at 250. Therefore, to save the computation time, a fixed grid number of 425,000 was used in this study. The convergence criterion of the flow field was set at 10^{-7} for the summation of the residuals.

The upper bound of the particle Reynolds number was set at Re_p = 250 since the flow over the particle becomes unsteady when Re_p > 250 as observed by Mochizuki (1961), while the upper bound of the plate Reynolds number was set at about 500,000 when the transition takes place from the laminar to turbulent flow as reported by Munson *et al*. (2006).

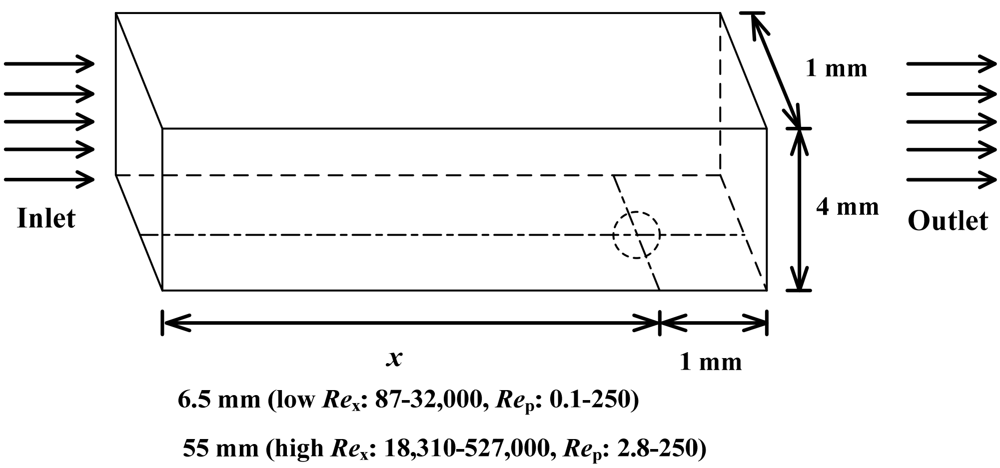

Fig. 1. Schematic diagram of the computational domain for the flow over a spherical particle attached on the flat surface.

Sweeney and Finlay (2007) found that the drag coefficient is dependent on the plate Reynolds number. In their study, Re_x was increased from 32,400 to 500,000 at a fixed Re_p of 250 only and the drag coefficient was found to increase by 17.4%. In this study, the influence of Re_x was extended to different Re_p from 3 to 250.

RESULTS AND DISCUSSIONS

The present simulation was first validated by checking the drag coefficient for a particle suspended in the air. As shown in Fig. 2, the calculated drag coefficient is seen to agree well with Eq. (6) with the deviation smaller than 1.3–5.6% for $10 \le Re_p \le 200$, with the smallest error of 1.3% at $Re_p = 200$. It is seen that the numerical method is able to predict the drag coefficient very well.

For the spherical particle attached on the flat surface, Fig. 3 shows the comparison of C_d obtained from the present numerical simulation with that in Eq. (8) and previous studies at low Re_x range from 87 to 32,000. The simulated drag coefficient compares very well with the value of Eq. (8) and the empirical value of Sweeney and Finlay (2007), Eq. (3), for Re_p = 0.1–250. O'Neill's equation is valid only at low particle Reynolds number of less than 1.0 as expected. The drag coefficient of Eq. (8) is slightly higher than the present simulation because of the Re_x effect, which was found by Sweeney and Finlay (2007) at $Re_p = 250$ and will be elucidated further in the following discussion. In comparison, both O'Neill's equation and Eq. (5) used by Ibrahim *et al.* (2008) deviate from the present simulation very much when $Re_p > 1.0$.

As discussed previously, Re_x has an effect to increase the drag coefficient and such effect was investigated by increasing the Re_x or increasing the distance x at a fixed Re_p. In the high Re_x range of this study, x was fixed at 55 mm as shown in Fig. 1. When Re_p is increased, the corresponding Re_x will also be increased with the maximum value of Re_x = 527,000 when Re_p = 250. Fig. 4 shows the comparison of C_d of the present simulation with Eq. (8) at higher Re_x from 18,310 to 527,000 (corresponding Re_p = 2.8–250), and low Re_x from 87 to 32,000 (corresponding Re_p = 0.1–250). In the high Re_x range, when Re_p is 250, C_d of Eq. (8) is in very good agreement with the present numerical simulation results at Re_x = 527,000 (Re_p = 250) with the deviation of only 1.6%, which confirms the finding of Sweeney and Finlay (2007). At Re_p of 91 and Re_x of 256,320, C_d is also increased and the deviation is increased to 4.7%. When Re_p is less than 91, Re_x can't be

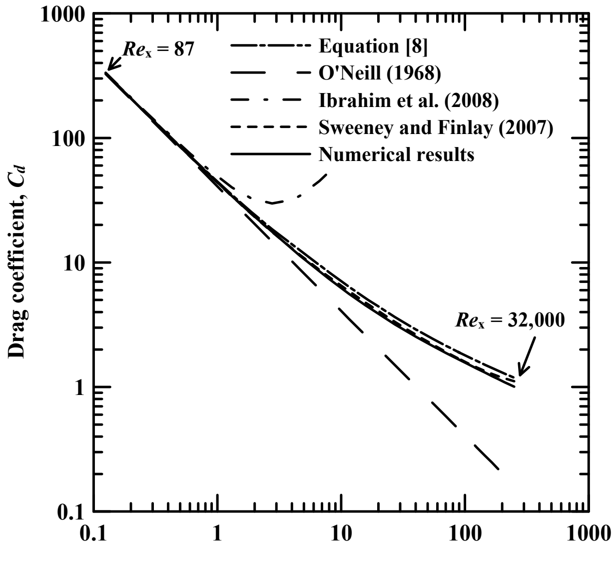

Fig. 3. Comparison of the drag coefficient of the present simulation with Eq. (8) and previous studies at low Re_x = 87–32,000.

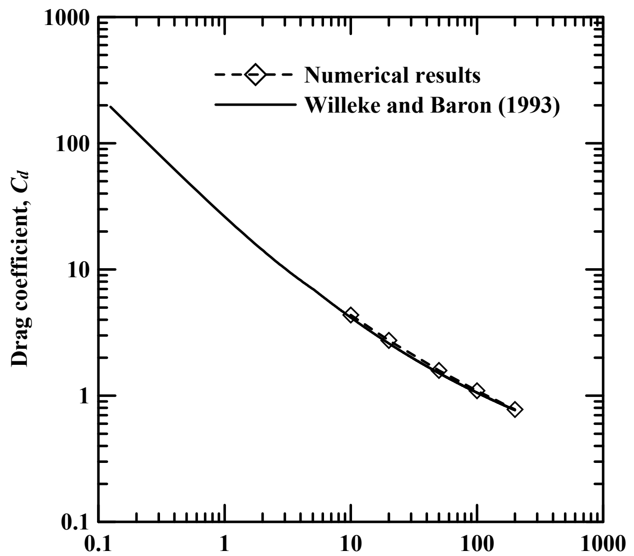

Fig. 2. Comparison of the calculated drag coefficient for a particle suspended in the air with the empirical values.

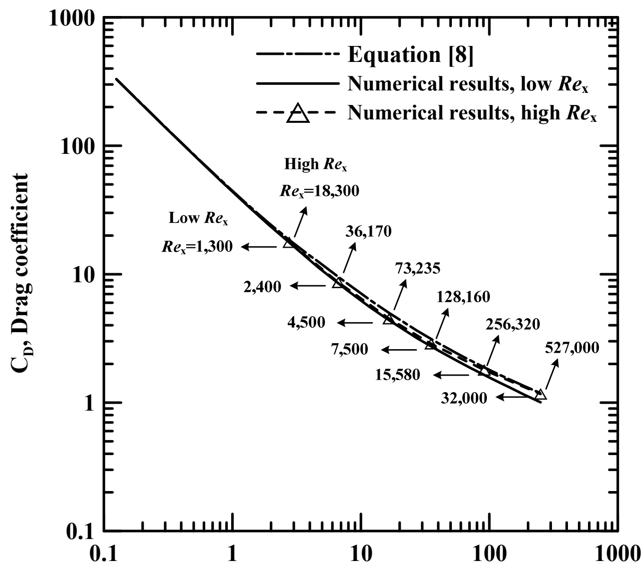

Fig. 4. Comparison of the drag coefficient of the present simulation with Eq. (8), effect of Re_x.

increased too much since the distance x will have to be extended to an unreasonable large number. Under this condition, Eq. (8) will overestimate the drag coefficient as shown in Fig. 4 but the maximum overestimation is less than 10.7% in the high Re_x range.

To find the applicable range of Re_x for Eq. (8), the effect of Re_x on C_d at Re_p = 250 was studied. As shown in Fig. 5, the simulated drag coefficient increases and approaches Eq. (8) as Re_x is increased from 20,000 to 527,000. The deviation is decreased from 22.7 to 1.6%. The reason why C_d decreases with decreasing Re_x can be seen in Fig. 6 where boundary velocity profiles at Re_x of 32,000, 145,000,

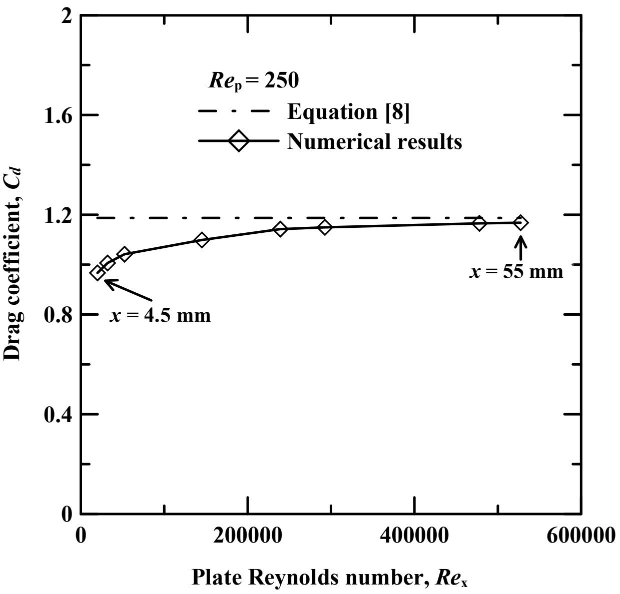

Fig. 5. Effect of the plate Reynolds number on the drag coefficient.

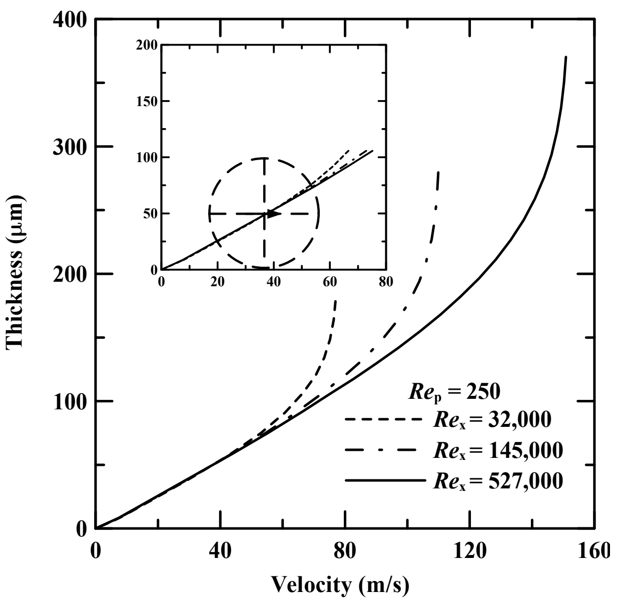

Fig. 6. Boundary layer velocity profiles at the plate Reynolds number of 32,000, 145,000 and 527,000, respectively, at Re_p = 250.

and 527,000, respectively, at Re_p = 250, are shown. It shows that velocity profile at the top half of the particle becomes steeper as Re_x is decreased from 527,000 to 32,000. This means that the velocity on the top half of the particle is reduced leading to the reduction of the drag coefficient as Re_x is reduced.

CONCLUSION

The previous analytical solution for C_d for a spherical particle attached on the flat surface, which was derived by O'Neill (1968), is only valid in the creeping flow conditions. In this study, C_d was calculated numerically to cover a wide Re_p range of 0.1 to 250. For a particle suspended in the air, an empirical drag coefficient exists, which is defined as $C_d = f \times 24/Re_p$, where f is a correction factor depending on Re_p. This correction factor f was checked by the present numerical simulation and found to be correct. The simulated drag coefficient for the particle attached on the flat surface also agrees well with the empirical drag coefficient proposed by Sweeney and Finlay (2007) for $0.1 \le Re_p \le 250$. It was found that the correction factor f for the suspended particle can also be used to correct for O'Neill's analytical equation to obtain C_d for the particle attached on the flat surface as Eq. (8) for Re_p = 0.1 to 250. At high Re_x from 18,310 to 527,000, the maximum error of Eq. (8) is only 4.7% and the effect of Re_x on C_d is small. At low Re_x from 87 to 15,580, the maximum error of Eq. (8) is less than 10.7% when Re_p < 100 and the effect of Re_x on C_d is also limited.

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