

On the Theory of Particle Cutoff Diameter and Collection Efficiency of Cyclones

Kung-Yu Kuo and Chuen-Jinn Tsai*

Institute of Environmental Engineering, National Chiao Tung University, No. 75, Poai St., 300 Hsin Chu, Taiwan, ROC

This study is aimed at investigating the accuracy of four existing theories on the cutoff diameter of cyclones, including Lapple (1950), Stairmand (1951), Barth (1956) and Iozia and Leith (1989), and six on the collection efficiency of cyclones, including Lapple (1950), Barth (1956), Leith and Licht (1972), Dietz (1981), Li and Wang (1989), and Iozia and Leith (1990). The predicted collection efficiency curves of the six different theories were compared with the experimental data in the literature. The comparison shows that different sets of experimental data agree better with a particular theory than the other theories and that none of the theories fits all the available experimental data perfectly well. The experimental data for the cutoff aerodynamic diameter, $D_{p_{a50}}$, were further plotted in the log-log coordinate of $\sqrt{CD_{p_{a50}}}/D$ versus Re_f and found to form a straight line (C: slip correction factor, D: cylindrical diameter of cyclone, Re_f : flow Reynolds number based on the radius of the cyclone minus the radius of the exit tube). In the plot, the experimental data were compared with four different theoretical predictions on the cutoff aerodynamic diameter. It was found that these four theories are more accurate when Re_f is between 10,000 and 100,000 and that none of them is better than the others.

Keywords: cyclone, collection efficiency, cutoff diameter

1. Introduction

A cyclone is a simple dust control device with low manufacturing and operating cost and is easy to maintain. It can be used to handle high temperature and high-pressure exhaust gas. The collection efficiency of a cyclone is better than that of a gravitational settling device, but is lower than that of a baghouse, venturi scrubber or electrostatic precipitator. The cutoff aerodynamic diameter of a cyclone is usually greater than 5 to 10 μm , and therefore it is often used as a precollector in front of other high-efficiency devices. Fig. 1 shows a

typical tangential-entry cyclone with various symbols indicated. The Stairmand-type high-efficiency cyclone was first mentioned by Stairmand (1951). It is one of the standard cyclone designs and is most commonly used. The relative dimensions of a Stairmand-type high-efficiency cyclone are $a=S=De=0.5D$, $b=0.2D$, $h=1.5D$, $H=4D$, and $B=0.375D$. (In Fig. 1, D: cyclone diameter; a: height of cyclone inlet; b: width of cyclone inlet; De: diameter of cyclone exit tube; S: length of vortex finder; h: height of cyclone cylinder; H: total height of cyclone; B: diameter of bottom of cyclone)

Literature is abundant on the experimental as well as theoretical work related to the particle collection efficiency or cutoff aerodynamic diameter of cyclones. The experimental data were often obtained under different operating conditions

*Corresponding author:

Tel.: +886-3-573-1880

Fax: +886-3-572-7835

E-mail address: cjtsai@green.ev.nctu.edu.tw

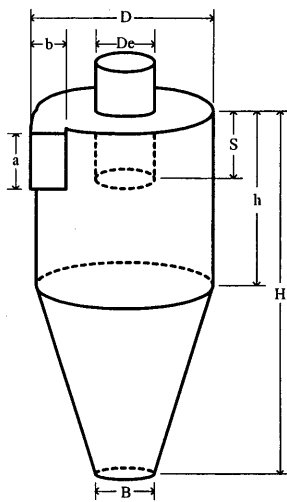


Figure 1. Typical tangential-entry cyclone with symbols indicated.

and using different cyclone geometries and dimensions. It is of great interest to know if the experimental data in the literature can be correlated in one simple equation to facilitate the design of a cyclone. The existing theories often do not agree well with all experimental data. It is of importance to know which of these theories can be applied and under what conditions.

2. Previous Work on the Cutoff Diameter and Collection Efficiency

Theoretical Cutoff Diameter

The cutoff diameter of the cyclone, Dp_{50} , is defined as the particle diameter corresponding to 50% collection efficiency. It is an indicator of the size range of particles that can be collected. When a particle undergoes a radial displacement equal to the half width of the cyclone inlet during the residence time, then 50% of the particle will be collected (Lapple, 1950). The cutoff diameter derived by Lapple (1950) is as:

$$Dp_{50} = 3 \sqrt{\frac{\mu b}{2\pi\rho_p U_i C N_t}} \quad (1)$$

where U_i is the gas velocity at the inlet; μ is the air dynamic viscosity; ρ_p is the particle density; C is

the slip correction factor of the particle corresponding to Dp_{50} . The number of turns N_t can be calculated as $N_t = tU_i/\pi D$ and the residence time t is equal to the volume of the cyclone divided by the volumetric flow rate, Q .

Stairmand (1951) and Barth (1956) both thought that when 50% of the particles spinning at the outer rim of the central core, where the tangential velocity is at its maximum (U_{tmax}), are collected, the corresponding Dp_{50} can be derived as:

$$Dp_{50} = \frac{3}{U_{tmax}} \sqrt{\frac{\mu Q}{\pi\rho_p C(H-S)}} \quad (2)$$

Barth (1956) suggested that the diameter of the central core is De while Stairmand (1951) claimed it to be $De/2$. Stairmand (1951) further added a friction loss factor in the denominator of Eq. (2) to modify Dp_{50} .

Iozia and Leith (1989) measured the tangential velocity profile of the gas within a pilot-scale cyclone and developed a new equation for Dp_{50} as

$$Dp_{50} = \frac{3}{U_{tmax}} \sqrt{\frac{\mu Q}{\pi\chi_c \rho_p}} \quad (3)$$

where

$$U_{tmax} = 6.1 \times U_i \times \left(\frac{ab}{D^2}\right)^{0.61} \times \left(\frac{De}{D}\right)^{-0.74} \times \left(\frac{H}{D}\right)^{-0.33}$$

and χ_c is the length of the central core.

Empirical Correlation of Experimental Data on Cutoff Diameter

Because of the differences in the size and geometry of the cyclones and the operation conditions, the range of the Reynolds number for the cyclone in different laboratory studies is quite different. Various dimensionless parameters have been used to correlate experimental data on the

cutoff diameter or collection efficiency, such as the Stokes number (S_{tk}) of Blachman and Lippmann (1974), the cyclone Reynolds number (Re_c) of Beeckmans and Kim (1977), the outlet tube Reynolds number (Re_o) of Saltzman and Hochstrasser (1983), and the inertial separation parameter (Ψ_A) of Bürkholz (1985). These parameters are defined as follows:

$$S_{tk} = \frac{C\rho_p Dp_a^2 U_i}{9\mu D} \quad (4)$$

$$Re_c = \frac{\rho U_i D}{\mu} \quad (5)$$

$$Re_o = \frac{4\rho Q}{\pi\mu De} \quad (6)$$

$$\Psi_A = \frac{3}{2} S_{tk} Re_o^{1/2} \zeta^{2/3} \quad (7)$$

where Dp_a and ρ are particle aerodynamic diameter and air density, respectively; $\zeta = \Delta P / (\rho U_i^2 / 2)$, where ΔP is the pressure drop through the cyclone.

The work of Moore and McFarland (1990) involved tests of four different sizes of Stairmand-type cyclones over four different flow rates and the cyclone flow Reynolds numbers (Re_c) ranged from 2,100 to 64,000. Their experimental data of Stokes number for the aerodynamic cutoff diameter, S_{tk50} , were fitted to a quadratic logarithmic function as

$$S_{tk50} = \frac{25,200}{Re_c^{2.72 - 0.119 \ln Re_c}} \quad (8)$$

Overcamp and Scarlett (1993) plotted the diagram of S_{tk50} versus Re_c . For each cyclone model they obtained a consistent curve with some scatter. However, when all cyclone models were pooled, the scatter was so large that the trends visible in the plots for the individual models

disappeared. Moore and McFarland (1993) fitted all experimental data in the Dp_{a50} - Re_f log-log plot by linear regression and found a good correlation existed. The flow Reynolds number, Re_f , is defined as:

$$Re_f = \frac{\rho(D - De)U_i}{2\mu} \quad (9)$$

The correlation coefficient can reach 0.9 and even as high as 0.99 in some experiments. Moore and McFarland (1996) further obtained a better correlation of experimental data in the log-log plot of Ψ_{50} versus Re_f , which was also confirmed by Lidén and Gudmundsson (1997). The dimensionless cutoff aerodynamic diameter, Ψ_{50} , is defined as

$$\Psi_{50} = \frac{\sqrt{C} Dp_{a50}}{D} \quad (10)$$

Theoretical Particle Collection Efficiency

Particle collection efficiency, η , is defined as the percentage of particles in number collected by the cyclone over the total number of particles entering the cyclone. Lapple (1950) found a correlation between the collection efficiency and Dp/Dp_{50} , which was shown by Theodore and Depaola (1980) to be:

$$\eta = \frac{1}{1 + (Dp_{50}/Dp)^2} \quad (11)$$

where Dp_{50} is calculated by Eq. (1).

Barth (1956) proposed an empirical correlation for efficiency based on the experimental results of several cyclone designs as:

$$\eta = \frac{1}{1 + \left(\frac{\pi h^* v_t^2 \rho_p Dp^2}{9\mu Q} \right)^{-3.2}} \quad (12)$$

where v_t is the tangential gas velocity at the edge of the central core, and h^* is the height of the central core.

Leith and Licht (1972) recognized the inherently turbulent nature of cyclone flow and considered an average residence time of particles within the cyclone. The assumption was made that the gas and particles are uniformly mixed across any cross section in the cyclone and that they become progressively cleaned as the exit is approached. The equation proposed by Leith and Licht is:

$$\eta = 1 - \exp\left[-2(C_g \psi)^{2n+2}\right] \quad (13)$$

where C_g is the dimension factor of a cyclone, ψ is the impaction parameter (ψ equals $S_{tk}/2$), and n is the vortex exponent (Alexander, 1949).

Clift et al. (1991) claimed that the above equation underestimated the mean residence time in the collection region by a factor of two. They derived a different collection efficiency equation but found that it was still under-predicted. Based on ter Linden's (1945) experimental investigations, Dietz (1981) divided the cyclone into three distinct flow regions: an entrance region, a downflow (annular) region and an upflow (core) region. Turbulence was assumed to provide uniform particle concentration profiles radially in each region. The equation for the collection efficiency was derived as:

$$\eta = 1 - \left[K_0 - \sqrt{K_1^2 + K_2} \right] \times \exp\left[\frac{-\pi D U_{pw} (S - a/2)}{Q} \right] \quad (14)$$

where U_{pw} is the velocity of particles near the cyclone wall. Dietz showed that this theory is a reasonable fit to the Stairmand's data (1951). However, Clift et al. (1991) also showed that this theory has some flaws, both in the physical interpretation of the particle transport phenomena and the mean residence time.

Li and Wang (1989) developed a new mathematical model to describe particle motion in the cyclones. They neglected turbulent diffusion throughout the interior of the fluid in the cyclone,

as well as the particle settling velocity in the z direction. Both turbulent diffusion of particles and particle bounce or reentrainment on the cyclone wall were considered. They derived the cyclone collection efficiency as:

$$\eta = 1 - \exp\left[-\lambda \frac{2\pi(S+L)}{a} \right] \quad (15)$$

where λ is a characteristic value, and L is the natural length of cyclone defined by Alexander (1949) as the farthest distance the spinning gas extends below the gas outlet duct, $L = 2.3De(D^2/ab)^{1/3}$.

Iozia and Leith (1990) correlated the collection efficiency using experimental data as:

$$\eta = \frac{1}{1 + (Dp_{50}/Dp)^\beta} \quad (16)$$

where Dp_{50} is calculated by Eq. (3), and β can be calculated from the following equation:

$$\ln \beta = 0.62 - 0.87 \ln(Dp_{50}) + 5.21 \times \ln\left(\frac{ab}{D^2}\right) + 1.05 \left[\ln\left(\frac{ab}{D^2}\right) \right]^2 \quad (17)$$

3. Comparison Between Theory and Experimental Data

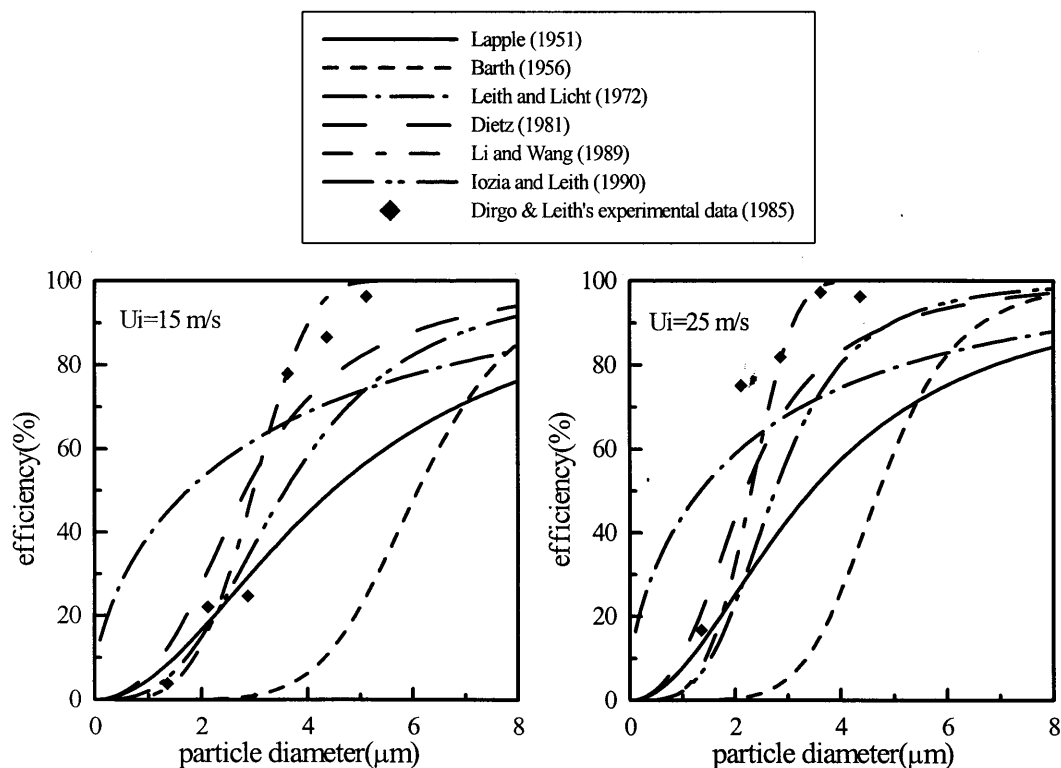
Particle Collection Efficiency Curve

In this study, eight groups of experimental data were used, including those of Dirgo and Leith (1985), Iozia and Leith (1989, 1990), Kim and Lee (1990), Lin (1996), Hsiao (1997), Moore and McFarland (1993), Tsai et al. (1999), and Zhu and Lee (1999). As shown in Table 1, the ranges of both the cyclone diameter and operating conditions are wide in these experiments. The diameter of the cyclone ranges from 1 to 30.5 cm and the flow Reynolds number ranges from 410 to 122,200.

The comparison between three groups of experimental data on the particle collection

Table 1. List of cyclone size and operating conditions of eight sets of experimental data.

Authors	Cyclone diameter (cm)	Flowrate (lpm)	Inlet velocity (m/sec)	Cutoff aerodynamic diameter (μm)	Re_f ($\times 10^3$)	Ψ_{50} ($\times 10^{-5}$)
Dirgo and Leith (1985)	30.5	2791-13954	5-25	1.78-5.94	24.4-121.9	0.61-1.97
Kim and Lee (1990)	2.19-4.11	8.8-18.4	2.04-4.26	2.87-8.41	0.41-4.27	8.86-38.77
Lin (1996)	13.3	1157-2759	10.9-26	1.45-2.36	23.3-55.62	1.15-1.83
Tsai et al. (1999)	1-1.8	1.7	5.85	3.45-3.91	0.94-1.69	22.2-35.2
Hsiao (1997)	1.8	1.5-2.1	5.17-7.23	3.44-4.28	1.50-2.09	19.5-24.2
Iozia and Leith (1989, 1990)	25	2820-8520	7.52-30.4	2.08-4.59	30.2-122.2	0.86-1.87
Moore and McFarland (1993)	3.81-8.89	16.3-124	0.83-4.05	4.0-18.8	0.69-5.23	6.05-30.65
Zhu and Lee (1999)	3.05	60-110	13.4-24.6	0.3-2.6	6.57-12.05	1.22-8.78

**Figure 2.** Comparison of Dirgo and Leith's (1985) experimental data with six theories on particle collection efficiency.

efficiency using the six theories outlined above is shown in Figs. 2-4. Fig. 2 shows that the experimental data of Dirgo and Leith (1985) for the relatively large cyclone ($D=30.5$ cm) agree best

with the predictions by Li and Wang (1989), while Fig. 3 shows that the experimental data of Kim and Lee (1990) for the relatively small cyclone ($D=2.19$ to 4.11 cm) agree best with the predictions by Barth

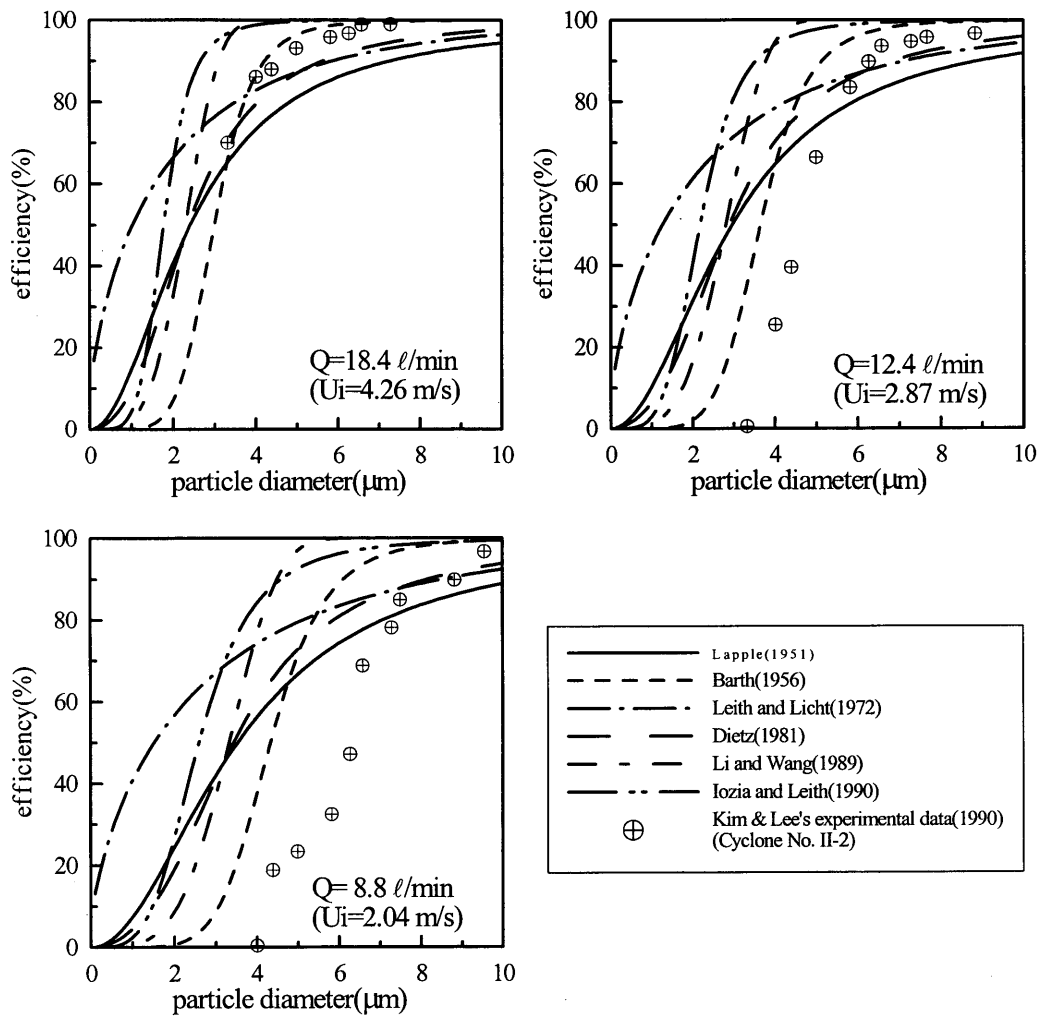


Figure 3. Comparison of Kim and Lee's (1990) experimental data with six theories on particle collection efficiency.

(1956) in most cases. For the middle cyclone diameter of 13.3 cm, the experimental data of Lin (1996) agree best with the predictions by Dietz (1981) in most cases. That is, different sets of experimental data agree better with a particular theory than the other theories, and none of the theories for the collection efficiency fit all experimental data perfectly well.

Cutoff Aerodynamic Diameter

To further test the accuracy of different theories, the method of Moore and McFarland (1996) was used to correlate the experimental dimensionless

cutoff aerodynamic diameter, Ψ_{50} (defined in Eq. (10)), with the flow Reynolds number, Re_f . Fig. 5 shows the seven groups of data, except those of Zhu and Lee (1999), and the resulting regression equation is

$$\ln \Psi_{50} = -0.74 \ln Re_f - 3.17 \quad (18)$$

The coefficient of correlation r^2 is as high as 0.98 covering a very wide range of Re_f . When Zhu and Lee's data were added, r^2 was not as good since their experiment mainly focused on the effect of the cylinder height and the length of exit tube on the collection efficiency and their cyclone geometry could be quite different from others. Moore and

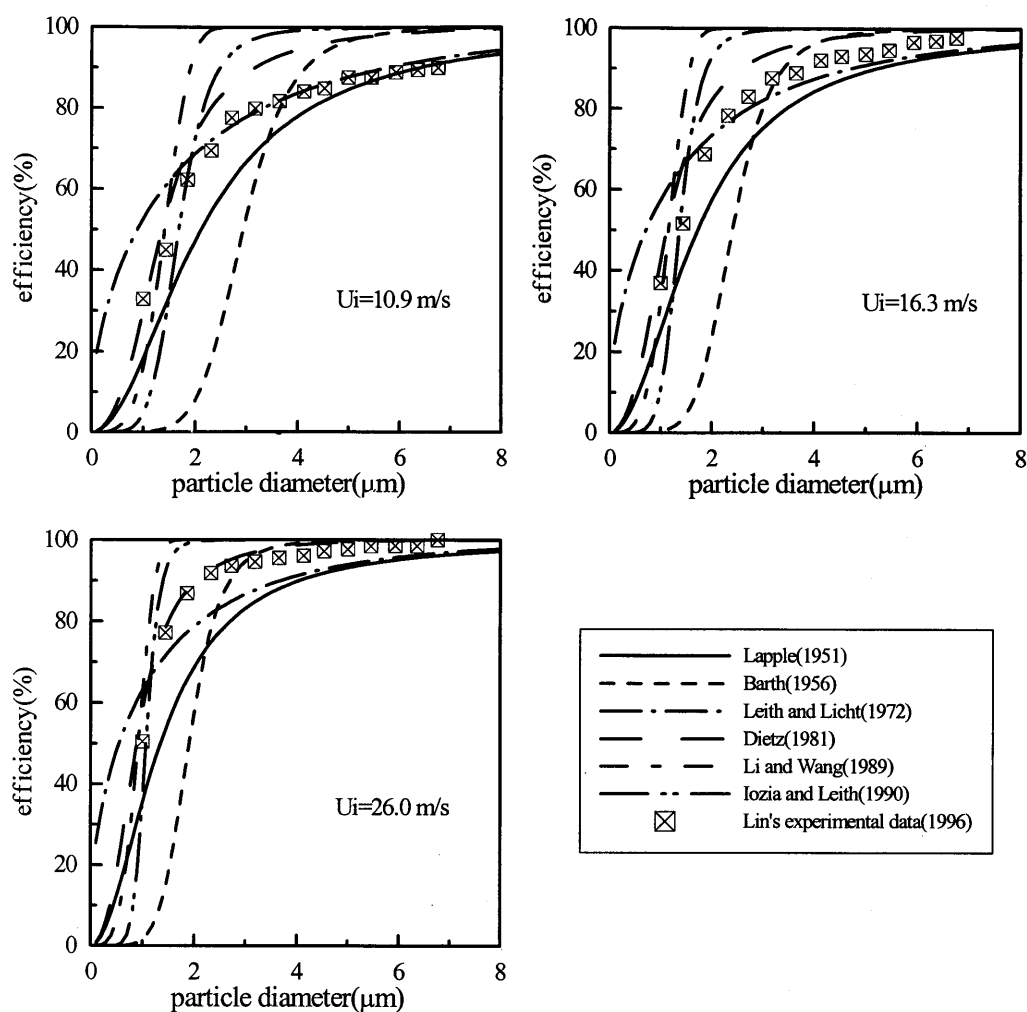


Figure 4. Comparison of Lin's (1990) experimental data with six theories on particle collection efficiency.

McFarland (1996) also obtained different regression equations for two different heights ($H/D = 2$ and 4) of cyclone. Therefore it is expected that cyclones of the same geometry should have the best-correlated equation for Ψ_{50} versus Re_f .

This is indeed the case as shown in Fig. 6 where only the experimental data of the Stairmand-type cyclones were selected from the eight experimental groups, and the resulting regression equation is

$$\ln \Psi_{50} = -0.72 \ln Re_f - 3.46 \quad (19)$$

The coefficient of correlation r^2 is as high as 0.994. It is recommended that Eq. (19) be used in the design of a Stairmand-type cyclone for the cutoff aerodynamic diameter. In the following, the fitted

straight line is used to compare the theoretical predictions of the cutoff aerodynamic diameter in the log-log plot of Ψ_{50} versus Re_f , either by the four theories on the cutoff diameter, Eqs. (1) to (3), or by the four theories on the collection efficiency, Eqs. (12) to (15).

Suppose that two Stairmand-type cyclones with a diameter of 5.2 cm and 19.2 cm, respectively, are operated at 20 °C and 1 atm. The gas inlet velocity of the former cyclone is assumed to be from 0.76~10.4 m/s and that of the latter from 2.8 ~38.4 m/s to cover a wide range of Re_f from 410 to 122,200. The particle density is assumed to be 0.98 g/cm³. The theoretical predictions of cutoff diameter by the four theories on the cutoff diameter and by the four theories on the collection efficiency

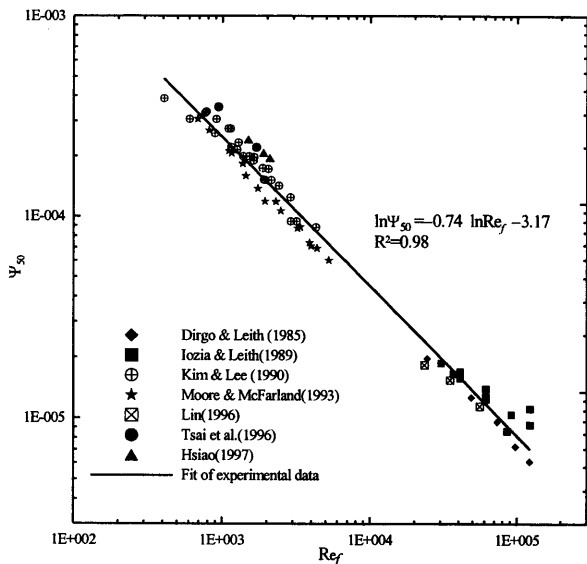


Figure 5. Plot of Ψ_{50} versus Re_f using all experimental data.

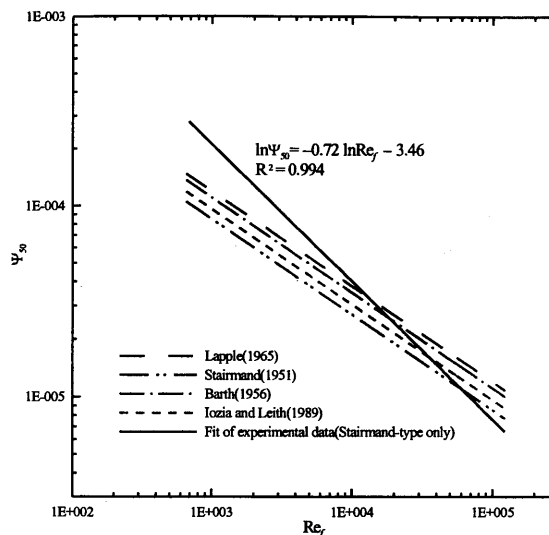


Figure 7. Comparison of predictions on cutoff diameter by four theories on the cutoff diameter in the Ψ_{50} - Re_f plot with the straight-line fit of the experimental data.

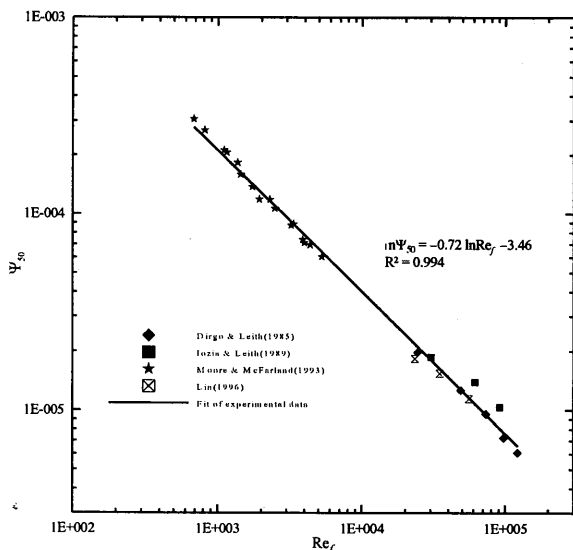


Figure 6. Plot of Ψ_{50} versus Re_f using the experimental data for Stairmand-type cyclones only.

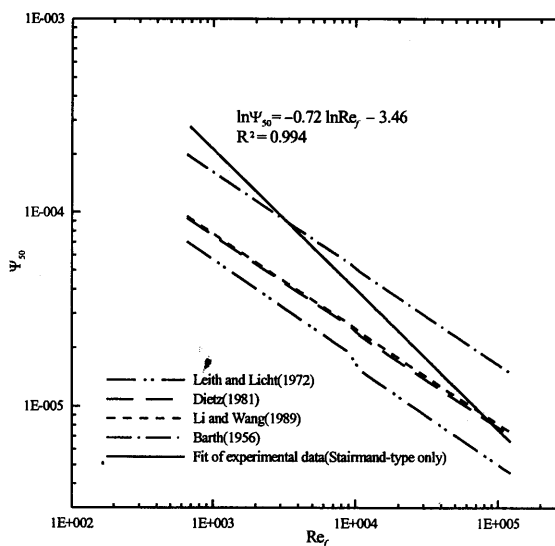


Figure 8. Comparison of predictions on the cutoff diameter by four theories on the collection efficiency in the Ψ_{50} - Re_f plot with the straight-line fit of experimental data.

are compared with the regression equation, Eq. (19), and plotted in Ψ_{50} - Re_f plot in Figs. 7 and 8, respectively. It can be seen that none of the theories are accurate in the entire range of Re_f .

In particular, the slopes of the predictions of the cutoff aerodynamic diameter in regression equations for all theories are about 0.5, which is much less than the experimental value of 0.72. All theories on the cutoff diameter under-predict Ψ_{50} when Re_f is less than 10,000 as shown in Fig. 7. The theories of Lapple (1965) and Barth (1956)

predict Ψ_{50} more accurately when Re_f is about 15,000 to 20,000 than at other Re_f while the theory of Stairmand (1951), and Iozia and Leith (1989) are more accurate at higher Re_f .

The dimensionless cutoff aerodynamic diameters, Ψ_{50} , are calculated from four theories on the particle collection efficiency and compared with the experimental correlation, Eq. (19), as shown in Fig. 8. It is seen that three theories, Dietz (1981), Leith and Licht (1972) and Li and Wang (1989) all

under-predict Ψ_{50} in the entire range of Re_f . The theory of Barth (1956), Eq. (12), is more accurate at Re_f of about 3,000 than the other Re_f .

4. Conclusion

There are many parameters to be considered when designing a cyclone. These include the type and geometry of the cyclone, flow rate and inlet geometry. Many factors which influence the collection efficiency remain to be resolved, including the flow interference at the inlet, particle bounce and reentrainment from the cyclone wall, diffusion deposition of particles, the effect of particle loading, coagulation of particles and the effect of surface roughness of the cyclone wall. All these factors render the theoretical analysis difficult. This study shows that a theory on the collection efficiency or cutoff aerodynamic diameter may only be accurate for a particular cyclone size or flow Reynolds number. None of the theories are accurate in the entire range of flow Reynolds number.

In the aspect of experimental data, this study shows that all experimental data are well correlated as a straight line in the log-log plot of Ψ_{50} versus Re_f for the Stairmand-type cyclones. The correlated equation can be used to calculate the cutoff diameter accurately over a wide range of Re_f for the Stairmand-type cyclones.

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