A Mathematical Study of a Fire in the Interaction Zone Between Flows with Different Velocities

David Katoshevski*

Department of Environmental Engineering,
Ben-Gurion University of the Negev, Beer-Sheva 84105, Israel

Abstract

An analytical study is presented for describing a flame in a shear layer flow formed between a gaseous fuel stream and an oxidizer stream, moving at different velocities. The diffusion flame-sheet approximation is addressed. It is shown how the shear layer flow-field driven by various ratios of free- flows velocities influences flame properties. The role of the "equivalence ratio for diffusion flames", that is, the product of stoichiometric ratio and the concentration ratio of fuel/oxidizer at the outer flows, is analyzed, in terms of flame shape and location. Flame shapes regimes are described in terms of equivalence ratio and velocity ratio. A "turning point" is revealed in the shift of the flame location from one stream towards the other with increasing Schmidt number. The value of the corresponding "turning point" equivalence ratio, in which the flame shift changes direction, is found to be governed by the velocity profile and specifically by the free stream velocity ratio. Moreover, this ratio is shown to control also the sensitivity of the flame location to changes in the value of the Schmidt number. Downstream velocity deceleration is also addressed, with respect to flame location and flame shape, showing a shift of the flame towards the fuel stream and a change in flame curvature. This study of the location and shape of such a flame configuration elucidates the ways these flame characteristics may be manipulated. It also points out the general region of the main production of air-pollutants in related combustion cases which exist in industry and in the outdoor-atmosphere where fire is occurring between two flows of different chemical species moving at different velocities.

Keywords: Shear Flow, Flame, Analytical Solution.

* Corresponding author. Tel: 972-8-6479094; Fax: 972-8-6472983
E-*mail address*: davidk@bgu.ac.il

INTRODUCTION

When a shear layer is formed between two streams, one, a gaseous fuel stream and the other an oxidizer, a diffusion flame may occur under the appropriate conditions. A similar statement was made by Burke and Schumann in their 1928 work that analyzes a diffusion flame in the case of two concentric streams flowing at the same constant velocity. It is a well known fact that the pollutants emissions from a fire reach their maximum levels in the area where the temperature is maximum, or in other words, where the main reaction takes place, and hence the study of both the location and the shape of the flame and the parameters which influence them will help to predict the area of high pollutants concentrations. Hence, the aim here is to mathematically analyze the behavior of the location of the flame and its shape, in response to: 1) Changes in free stream velocity ratio; 2) Downstream velocity deceleration; 3) Fuel to oxidizer various ratios; and 4) Fuel and oxidizer Lewis numbers. The analysis presented here is entirely mathematical besides some auxiliary numerical calculations.

A flame in a shear layer occurs in various situations in industry and in the outdoor-atmosphere. In terms of research this configuration is attractive due to the fact that, in spite of being physically two dimensional, it can be reduced to a one dimensional problem, by an appropriate similarity transformation, whilst at the same time preserving the essential features of the combustion. Analysis of this system can lead to conclusions that are applicable in more complex systems, in a manner analogous to the way that the study of flames in a laminar stagnation flow is the basis for the turbulent situation (Peters, 1984).

The concept of the diffusion flame has been discussed extensively in the literature, by Greenberg (1989), Kuo (1986), and others (Allison and Clarke, 1980; Ishizuka, 1984; Chung and Law, 1984, Tambour and Katoshevski, 1994; Li et al., 1995; Daou and Linan, 1998; Katoshevski and Tambour, 2000) to cite just a few contributions. We will approach this special case from a different perspective focusing mainly on the location and shape of the flame. With respect to flame location an interesting feature is revealed in which its shift with the Schmidt number switches direction at a certain equivalence ratio depending on the velocity ratio.

The profound role played by the free stream velocity ratio in terms of flame characteristics did not gain a significant attention in the literature. Accounting for downstream pressure gradient has inspired the work by Marble and Hendricks (1986) who analyzed the case where the two streams move at the same velocity which is decreasing downstream.

In the present work we thus focus on the influence of the subtleties of the shear layer flow field and chemical equivalence ratio on flame characteristics.

PROBLEM FORMULATION, AND FLOW-FIELD SIMILARITY EQUATIONS

A two dimensional steady laminar shear flow is considered. It is formed by two gaseous streams moving at different velocities, where one is basically the combustible species and the other, an oxidizer, in addition to an inert gas. Under the appropriate operating conditions a diffusion flame may occur in such a configuration, as described schematically in Fig. 1. For the frame of the current investigation, the chemical reaction is assumed to occur in a narrow flame sheet, and can be described by a global one-step kinetic procedure, that is, a representative single reaction (Tambour and Katoshevski, 1994). We consider the velocities to be small compared to the speed of sound, neglect viscous dissipation, radiation, the work done by the pressure, and the Dufour and Soret effects.

As shown in Fig. 1, the fuel stream is denoted by “I” and the oxidizer stream by “II”. The longitudinal and lateral directions are denoted by x and y, respectively and we distinguish here between the longitudinal free stream velocity of the fuel stream U_I and that of the oxidizer stream U_{II}, where

$$U_I = \hat{U}_I x^{\alpha} \quad ; \quad U_{II} = \hat{U}_{II} x^{\alpha} \tag{1)-(2}$$

x is a normalized longitudinal distance, and $\hat{U}_I$ and $\hat{U}_{II}$ are characteristic longitudinal velocities for streams “I” and “II”. At present we consider constant–velocity or decelerating flows, that is the power α is zero or negative (see also Katoshevski et al. 1993; Katoshevski and Tambour, 1994). However, the formulation is general and can handle downstream acceleration as well, that is, $\alpha>0$.

The Howarth boundary layer similarity formulation with longitudinal pressure gradient is performed, for the gas phase momentum equations. Defining a stream function ψ and similarity variable η,

$$\Psi = \left[\frac{1}{2}(\alpha+1)\right]^{-1/2} x^{(\alpha+1)/2} f(\eta) \quad ; \quad \eta = \left[\frac{1}{2}(\alpha+1)\right]^{1/2} x^{(\alpha-1)/2} \int_0^y \frac{\rho}{\rho_\infty} dy \tag{3)-(4}$$

first leads to the normalized similarity function for the velocity profile

$$\frac{u}{U_I} = f'(\eta) \tag{5}$$

where the prime denotes differentiation with respect to η, that is $f'=df/d\eta$. Employing the Howarth transformation on the momentum equations leads to the following equation for the "upper" stream (stream "I" in Fig. 1)

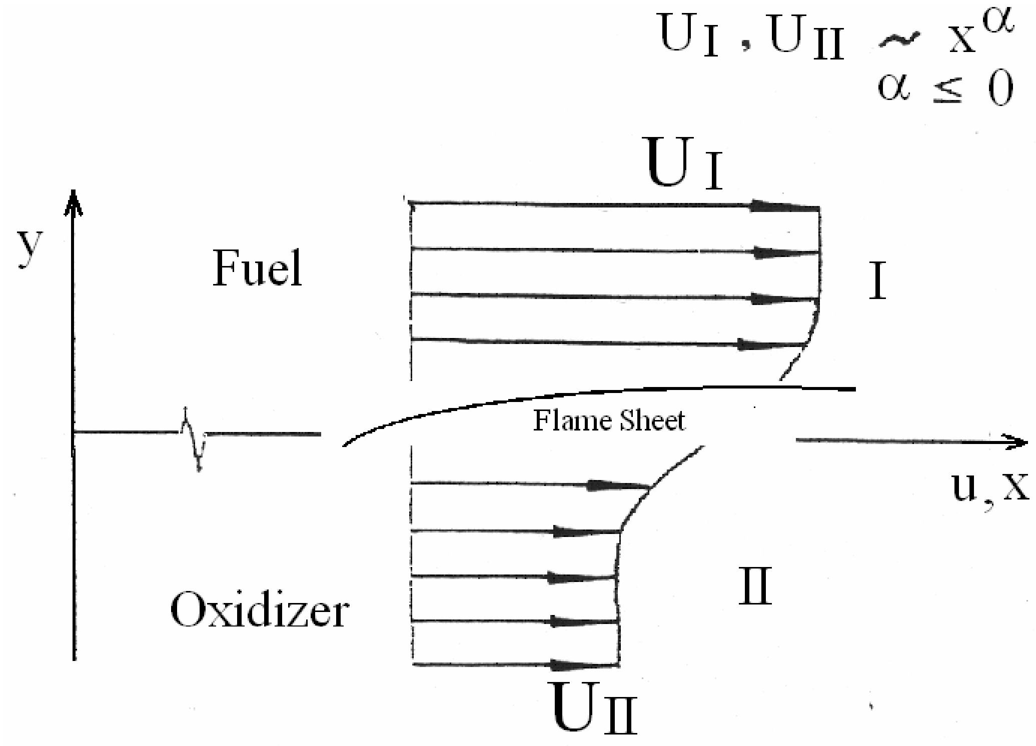


Fig. 1. Schematic description of the problem.

$$f''' + ff'' + \frac{2\alpha}{\alpha+1}\left[\frac{\bar{T}}{\bar{T}_\infty} - (f')^2\right] = 0 \tag{6}$$

and for the "lower" stream (stream "II"),

$$f''' + ff'' + \frac{2\alpha}{\alpha+1}\left[\frac{\bar{T}}{\bar{T}_{-\infty}}\left(\frac{U_{II}}{U_I}\right)^2 - (f')^2\right] = 0 \tag{7}$$

At the edges of the shear layer, the boundary conditions are given by

$$\eta \to +\infty: \; f'_{\infty} = 1 \tag{8}$$

$$\eta \to -\infty: \; f'_{-\infty} = U_{II} / U_{I} \tag{9}$$

At the interface, $\eta = 0$ there is continuity in terms of velocity and shear stress, that is, continuity of f', and $\sqrt{\rho\mu}\, f''$, respectively.

As shown later, the velocity ratio, U_{II} / U_{I} that determines the momentum boundary condition at the oxidizer stream, is a key parameter in determining the location of the flame and its shape.

In Eqs. (6)-(7) $\overline{T}$ denotes normalized temperature and it is divided in these equations by its boundary values at the two edges of the shear flow, thus this ratio becomes unity in both outer flows.

The above equations (Eq. (6)-(7)) for a shear layer with longitudinal pressure gradient and negative values of α, are discussed in Katoshevski et al. (1993), in the isothermal context. The solution of these equations has an entirely different behavior than that of the α=0 case studied by Lock (1951). Zero α value leads to a single solution for the momentum equations, while when this parameter is negative there is an infinite number of solutions that satisfy the boundary conditions. The solution that is chosen is the one that admits an exponential behavior of the decay of the velocity profile as it reaches the outer flows boundary conditions (Katoshevski et al. 1993). In the course of the current work we will analyze the base case of zero α and asymptotically examine the effect of a small deviation of the power α from the base case. For that purpose we represent the function *f* as

$$f = f^{(0)} + \alpha f^{(1)} + O(\alpha^2) \tag{10}$$

This leads, for both streams, to the following zero's order equation, O(1):

$$f^{(0)}{'''} + f^{(0)} f^{(0)}{''} = 0 \tag{11}$$

and the first order equation, O(α):

$$f^{(1)}{'''} + f^{(1)} f^{(0)}{''} + f^{(0)} f^{(1)}{''} + 2\left(g - \left(f^{(0)}{'}\right)^2\right) = 0 \tag{12}$$

Where,

$$g = \overline{T} / \overline{T}_{\infty} \tag{13}$$

for the upper stream, and for the lower one:

$$g = \left(U_{II} / U_{I}\right)^2 \overline{T} / \overline{T}_{\infty} \tag{14}$$

At the edges of the shear flow $f^{(1)}{}'_{\infty} = f^{(1)}{}'_{-\infty} = 0$, and $g_{\infty} = 1$, $g_{-\infty} = \left(U_{II} / U_{I}\right)^2$

SOLUTION FOR THE GOVERNING SIMILARITY EQUATIONS

In the current work we adopt the commonly used diffusion flame sheet concept in which the chemical reaction is taking place within a very narrow area. The above similarity transformation is employed on the equations for the temperature and species, which are also introduced as perturbed functions, similar to the above mentioned *f* function. Thus, for the normalized temperature, and for the fuel (sub-index *f*) and oxidizer (sub-index *o*) mass fractions we assume, respectively:

$$\overline{T} = \overline{T}^{(0)} + \alpha \overline{T}^{(1)} + O(\alpha^2) \tag{15}$$

$$m_f = m_f^{(0)} + \alpha\, m_f^{(1)} + O(\alpha^2) \tag{16}$$

$$m_o = m_o^{(0)} + \alpha\, m_o^{(1)} + O(\alpha^2) \tag{17}$$

where the O(α) terms vanish at the outer edges of the shear flow.

This leads to the following equations for the normalized temperature and for the fuel and oxidizer mass fractions, which are considered as functions of η, O(1):

$$\Pr^{-1} \overline{T}^{(0)}{}'' + f^{(0)} \overline{T}^{(0)}{}' = -QS_R\, \delta(\eta - \eta_F) \tag{18}$$

$$Sc_f^{-1} m_f^{(0)}{}'' + f m_f^{(0)}{}' = \nu_f M_f S_R\, \delta(\eta - \eta_F) \tag{19}$$

$$Sc_o^{-1} m_f^{(0)}{}'' + f m_f^{(0)}{}' = \nu_o M_o S_R\, \delta(\eta - \eta_F) \tag{20}$$

and for O(α):

$$\Pr^{-1}\, \overline{T}^{(1)}{}'' + f^{(0)}\, \overline{T}^{(1)}{}' + f^{(1)}\, \overline{T}^{(0)}{}' = 0 \tag{21}$$

$$Sc_f^{-1} m_f^{(1)}{}'' + f^{(0)} m_f^{(1)}{}' + f^{(1)} m_f^{(0)}{}' = 0 \tag{22}$$

$$Sc_o^{-1} m_o^{(1)}{}'' + f^{(0)} m_o^{(1)}{}' + f^{(1)} m_o^{(0)}{}' = 0 \tag{23}$$

In the above, Q denotes the normalized specific heat release, *Pr* is the Prandtl number, and Sc_f and Sc_o are the Schmidt numbers for the fuel and oxidizer, respectively. S_R is an Arrhenius-type of a reaction term based on a one step global chemical reaction scheme. The term $\delta(\eta - \eta_F)$ is a Dirac delta function which sets to zero the chemical source terms outside of the flame sheet that is located at $\eta = \eta_F$. This representation of the reaction implies that an explicit form for the reaction term plays no role, as will become clear in short.

The solution for the zero's order incorporates the following defined function,

$$F_i(\eta) = \frac{\int_{-\infty}^{\eta} \exp\left(-Sc_i \int_0^{\overline{\eta}} f^{(0)} d\eta\right) d\overline{\eta}}{\int_{-\infty}^{\infty} \exp\left(-Sc_i \int_0^{\overline{\eta}} f^{(0)} d\eta\right) d\overline{\eta}} \;;\; i = f, o, T \tag{24}$$

where for $i=T$ the Schmidt number *Sc* is replaced by the Prandtl number *Pr*.

For the species distributions one obtains,

$$m_f^{(0)}(\eta) = \left[\frac{F_f(\eta) - F_f(\eta_F)}{1 - F_f(\eta_F)}\right] m_{f\infty} \tag{25}$$

and

$$m_o^{(0)}(\eta)=\left[1-\frac{F_o(\eta)}{F_o(\eta_F)}\right]m_{o-\infty} \tag{26}$$

where $m_{f,\infty}$ and $m_{o,-\infty}$ denote the boundary conditions at the outer edges of the shear flow.

Each of the species exists only on one side of the flame sheet. Pollutants, which are products of the chemical reaction, will have a maximum peak of concentration at the flame location. As the flame location is determined here by the reactants, the pollutants' concentrations will not have an effect on that. η_F, as already mentioned, denotes flame location in terms of the similarity variable. This location is determined by

$$\left[\frac{1-F_f(\eta_F)}{F_o(\eta_F)}\right]\frac{F_o'(\eta_F)}{F_f'(\eta_F)}=\frac{Sc_o}{Sc_f}\phi \tag{27}$$

where ϕ is the product of the stoichiometric and concentration ratios at $\pm\infty$ (assuming non-zero oxidizer concentration), and is similar to the "equivalence ratio" used in other studies of diffusion flames (Allison and Clarke, 1980),

$$\phi=\hat{\nu}\frac{m_{f,\infty}}{m_{o,-\infty}}. \tag{28}$$

Next, in order to obtain the approximated flame temperature $\overline{T}_{\max}^{(0)}$ we first write the expressions describing the temperature distribution

$$\eta\geq\eta_F \quad \overline{T}^{(0)}(\eta)=\frac{F_T(\eta)-1}{F_T(\eta_F)-1}\left(\overline{T}_{\max}^{(0)}-\overline{T}_{\infty}\right)+\overline{T}_{\infty} \tag{29}$$

$$\eta\leq\eta_F \quad \overline{T}^{(0)}(\eta)=\frac{F_T(\eta)}{F_T(\eta_F)}\left(\overline{T}_{\max}^{(0)}-\overline{T}_{-\infty}\right)+\overline{T}_{-\infty} \tag{30}$$

and employ a jump condition for the temperature, leading to

$$\overline{T}^{(0)}_{\max} = F_T(\eta_F)\overline{T}_{\infty} + [1 - F_T(\eta_F)]\overline{T}_{-\infty} - \frac{F_f'(\eta_F)}{1 - F_f(\eta_F)} \frac{1 - F_T(\eta_F)}{F_T'(\eta_F)} F_T(\eta_F) \frac{1}{Le_f} \phi \frac{m_{o,-\infty}}{\nu^*} \ . \tag{31}$$

where $\nu^* = -\nu_o M_o / Q$.

What remains in order to complete this part of the solution is an expression for $F(\eta)$. This is possible in cases where the function $f(\eta)$, that is the solution for the momentum equations (Eqs. (6)-(7)), could be replaced by an explicit approximation. One such an example of an approximation is easily obtained in case where there is a negligible velocity difference between the two streams, that is, in the limit $U_{II}/U_I \to 1$. In this case, for a *qualitative* analysis (which will soon turn out as being valuable), one may assume a linear behavior for the function f in terms of η, and simply replace f by η. In the limit considered, $f = \eta$ is a solution for Eqs. (6) and 7 (which become identical), and satisfies the boundary condition for $f(\eta)$ at η=0, where $f(\eta=0)=0$ (Lock, 1951), and also satisfies the condition for its derivative at $\eta \to +\infty$, that is $f'(\eta \to +\infty)=1$. $F(\eta)$ is then evaluated as

$$F(\eta) \approx \frac{\int_{-\infty}^{\eta} exp\left(-\frac{Sc}{2}\overline{\eta}^2\right) d\overline{\eta}}{\int_{-\infty}^{\infty} exp\left(-\frac{Sc}{2}\eta^2\right) d\eta} = \frac{1}{2}\left(1 + erf\left[(Sc/2)^{1/2}\eta\right]\right) \tag{32}$$

Substituting this in Eq. (27), for equal Schmidt numbers of the two species, the flame location η_F is then found approximately from

$$erf\left[(Sc/2)^{1/2}\eta_F\right] = (1-\phi)/(1+\phi). \tag{33}$$

Such an approximation is valuable for a qualitative analysis of the behavior of the flame location with changes in the Schmidt number when $U_{II}/U_I \to 1$. It reveals an interesting feature, in which, as *Sc* increases, the flame can shift either “up” or “down” depending on the value of ϕ. A shift of a flame due to a change in diffusion rate and reactants' concentrations in the free flows is an expected phenomenon, but here we will show how the turning point of that shift is controlled by the velocity ratio. As will be shown next, the shift up and down of the flame exists, not only when the two streams move at about the same velocity, $U_{II}/U_I \to 1$, but even at the other extreme in terms of velocity ratio, that is at $U_{II} / U_{I} = 0$ where one stream is moving and the other is at rest.

After the zero-order of solution is obtained, the effect of low extent of downstream deceleration (or acceleration) can be obtained by solving simultaneously the set of equations (Eq. (12)) and (Eqs. (21)-(23)), which result in correction terms for the velocity, temperature and species distributions. This effect will be presented later after a further insight into analytical findings regarding the base case of constant flow velocities.

Analysis of the leading order solution: a diffusion flame in a shear flow without downstream velocity variation

We deal now with the situation where both streams move at constant velocities. It actually coincides with the zero-order solution mentioned above. This category of cases includes the classical shear layer, which is formed when one stream moves at constant velocity and the other is at rest. For all these cases α=0, and the momentum equations (Eqs. (6)-(7)) for both streams reduce to

$$f''' + f\, f'' = 0\,. \tag{34}$$

where here we omit for convenience the upper index "0" used earlier for the zero-order problem.

This equation enables the expression for $F(\eta)$ (Eq. (24)) to be simplified using the following relation arising from it

$$\int_0^{\eta} f(\bar{\eta})d\bar{\eta} = -\,ln\frac{f''(\eta)}{f''(0)} \tag{35}$$

which, when substituted into Eq. (24) leads to

$$F(\eta) = \frac{\int_{-\infty}^{\eta} (f'')^{Sc}\, d\bar{\eta}}{\int_{-\infty}^{\infty} (f'')^{Sc}\, d\bar{\eta}} \tag{36}$$

for same species Schmidt numbers. For unity Schmidt numbers this reduces to

$$F(\eta)=\frac{f'(\eta)-f'_{-\infty}}{f'_{\infty}-f'_{-\infty}} \tag{37}$$

where $f'_{-\infty}=U_{II}/U_{I}$. In the above mentioned classical case where stream "I" moves at constant velocity and stream "II" is at rest $f'_{-\infty}=0$ and Eq. (27) reduces even further to

$$F(\eta)=f'(\eta) \tag{38}$$

so that $F(\eta)$ is replaced here by the normalized velocity distribution.

Complete sets of numerical distributions for the normalized velocity $f'(\eta)$ for various constant-velocity shear layers, including the above case, were given by Lock in 1951. Thus, these sets of values can serve here, together with Eqs. (27) 27 and 38, for obtaining the flame location η_F in terms of the parameter ϕ, without any numerical effort, by

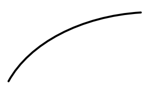
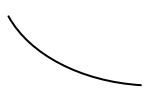
$$f'(\eta_F)=(1+\phi)^{-1} \tag{39}$$

The same applies to the flame shape. The value of η_F determines the flame shape in the x-y plane by Eq. (4), without considering density variations (Katoshevski and Tambour, 2000), where for α=0

$$y(x)_F=\eta_F\,(2x)^{1/2} \tag{40}$$

It should be noted that the interface between the two streams is assumed here at η=0 as was taken by Lock (1951) as well as by others, but this is not justified yet in the literature, and a shift along the η coordinate may still be feasible, unless it is determined by experimental tools.

Referring to Lock's work (1951), for the $U_{II}/U_{I}=0$ case, the similarity variable changes from negative to positive at f'=0.5873, so that the flame shape could be categorized using Eqs. (39)-(40) as follows, in terms of the equivalence ratio ϕ,

ϕ	Flame Shape	$y(x)_F$
< 0.703		$\sim +\sqrt{x}$
= 0.703		= 0 ; Flat Flame
> 0.703		$\sim -\sqrt{x}$

Now, after the location and shape of the flame in the above considered case are known, in order to complete the solution, $\overline{T}_{max}$ is calculated from Eq. (31), and the fuel and oxidizer profiles are readily obtained from Eqs. (25) and (26) as,

$$\eta \geq \eta_F \qquad m_f(\eta) = \left[\frac{f'(\eta) - f'(\eta_F)}{1 - f'(\eta_F)}\right] m_{f,\infty}; \qquad \overline{T}(\eta) = \frac{f'(\eta) - 1}{f'(\eta_F) - 1}\left(\overline{T}_{max} - \overline{T}_\infty\right) + \overline{T}_\infty \tag{41a,b}$$

$$\eta \leq \eta_F \qquad m_o(\eta) = \left[1 - \frac{f'(\eta)}{f'(\eta_F)}\right] m_{o,-\infty}; \qquad \overline{T}(\eta) = \frac{f'(\eta)}{f'(\eta_F)}\left(\overline{T}_{max} - \overline{T}_{-\infty}\right) + \overline{T}_{-\infty} \tag{42a,b}$$

Note that in the above terms the Schmidt numbers are unity. The corresponding terms for the case where $Sc_f \neq Sc_o$ (or $Le_f \neq Le_o$) could be deduced from Eqs. (24)-(26) and 29-30.

For the numerical representation we consider an n-decane – oxygen flame. The *base case* will be refereed here to $U_{II}/U_I = 0$, $Sc_f = Sc_o = 1$, $\phi = 0.703$ and $\eta_F = 0$, that is a flat shaped flame sheet. The effect of the velocity ratio U_{II}/U_I on the flame location and its shape will be discussed next, in the results and discussion section.

RESULTS AND DISCUSSION

In the course of the study we first start with the cases of constant stream velocities and later we will address the role of downstream deceleration. Hence, we will now show results for the constant velocities, starting with a base case of $Sc_f = Sc_o = Sc = 1$. When analyzing the parameters influencing the flame properties in this configuration, the effect of the velocity ratio U_{II}/U_I on the flame shape is found to be of high importance, and could be addressed by making use of Eqs. (27) and (37), which leads to

$$f'(\eta_F) = \left(1 + \frac{U_{II}}{U_I}\phi\right) / (1 + \phi) \tag{43}$$

If, for example U_{II}/U_I=1/2, according to Lock (1951) $f'(0)$=0.7657, so that a flat flame in this case corresponds to an equivalence ratio ϕ of 0.8818. For a velocity ratio U_{II}/U_I=1/4, $f'(0)$=0.6661, then ϕ=0.8025, and for U_{II}/U_I =3/4, $f'(0)$=0.8784 and ϕ=0.9470. Other combinations of U_{II}/U_I and ϕ leading to a flat shaped flame could be deduced from Fig. 2. This figure indicates the flame sheet shape regimes in the U_{II}/U_I - ϕ plane, and it could serve in pre-designing of experiments and facilities where a flame may exist between two streams moving at different velocities, when both Schmidt numbers are of the order of unity.

Next, we describe the case where $Sc_f = Sc_o = Sc \neq 1$. The effect of the Schmidt number, *Sc,* on the flame location, in terms of ϕ is shown in Fig. 3. In this case $U_{II}/U_I = 0$, that is when stream "I" is in motion and stream "II" is at rest. For a fixed Schmidt number, as expected, an increase of the equivalence ratio value ϕ, which either increases the fuel mass fraction at the "upper" stream $m_{f,\infty}$ or decreases the oxidizer mass fraction at the "lower" stream $m_{o,-\infty}$, would lead to a shift of the flame "down" towards the oxidizer stream, in order to reach the stoichiometric zone.

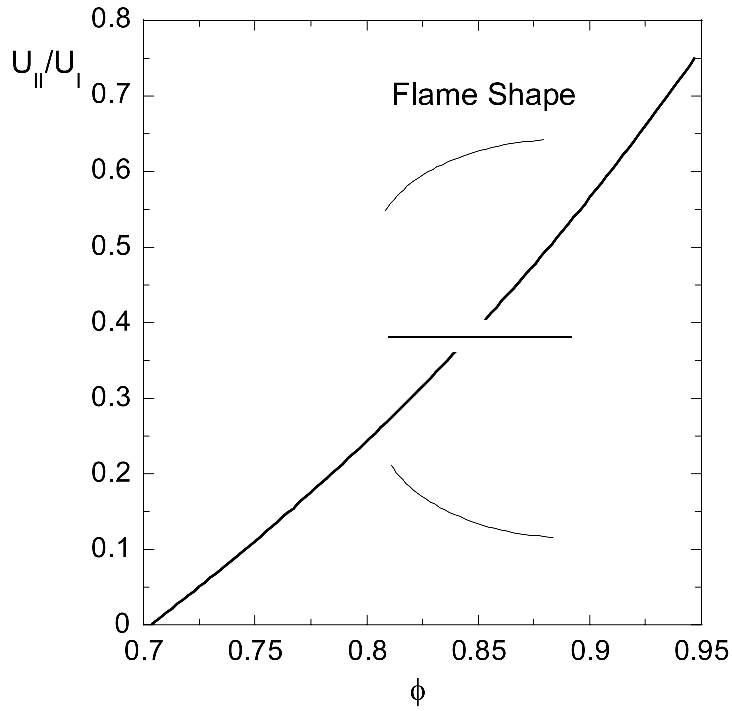


Fig. 2. Flame shapes regimes as a function of free streams velocity ratio U_{II}/U_I and equivalence ratio ϕ. $Sc_f = Sc_o$=1.

Interestingly, when the Schmidt number is changed, "a turning point" in the shift direction of the flame is revealed (see Fig. 3). In the lower range of ϕ values, the flame shifts downwards with an increase in *Sc*, up to a turning point ϕ_{tp} where the flame starts to shift upwards. In the figure, calculations for *Sc*=0.5 and *Sc*=2.0 are chosen arbitrarily, but calculations for other Schmidt number values as well, for example *Sc*=1.0 show that their lines cross also through the same turning point. The possible existence of such a turning point feature has been already indicated here mathematically by Eq. (33). This turning point of the equivalence ratio, ϕ_{tp}, is found to be governed by the velocity ratio U_{II}/U_{I} , as shown in Fig. 3. The impact of increasing the velocity ratio is two fold: (1) an increase of the ϕ_{tp} value; and (2) a reduction in the sensitivity to changes in the Schmidt number. At the limit, in which $U_{II}/U_{I} \rightarrow 1$, according to Eq. (33), for any *Sc* value ϕ_{tp} would equal unity. Eq. 33 shows also that for the latter limiting case, for a fixed ϕ value, an increase in *Sc* leads to a decrease in the absolute value of η_F, thus *Sc* serves as a stabilizer which in this case maintains the flame close to a flat one. In the other cases where the velocity ratio does not tend to unity, as in Fig. 3, the Schmidt number again serves as a stabilizer but in a broader sense which is to maintain the flame location within the interaction zone between the two streams, that is, within the shear layer. This figure also shows that for ϕ=1, and U_{II}/U_{I} close to 0.5, the location of the flame is in the vicinity of η=0, which is in a qualitative agreement with the findings of Mungal and Dimotakis (1984), even though in their case the flow was turbulent.

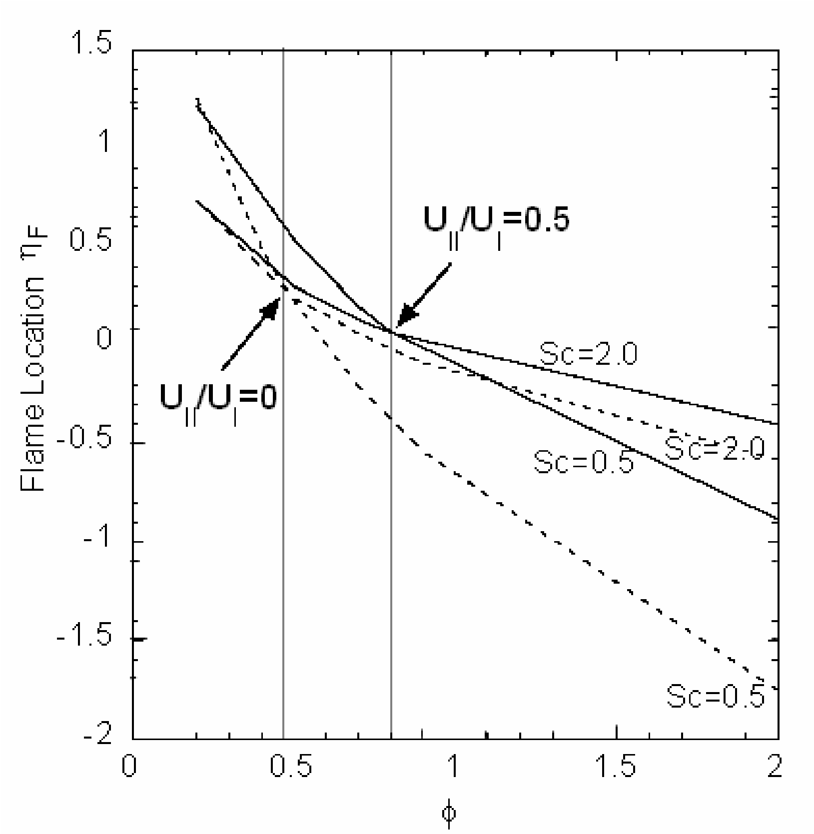


Fig. 3. Effect of the velocity ratio U_{II}/U_I on the "flame shift turning point" (marked by the arrows). Broken lines: $U_{II}/U_I = 0$, *Sc*=0.5, 2.0; Solid lines: $U_{II}/U_I = 0.5$, *Sc*=0.5, 2.0. Le_f=Le_o.

Finally, the $Sc_f \neq Sc_o$ case is described in Fig. 4 in terms of the Lewis numbers (*Le=Sc/Pr*). We first keep the oxidizer Lewis number as unity and use our base case ϕ value of 0.703, and describe the changes in flame location as a function of the fuel Lewis number. As the fuel Lewis number Le(Fuel) increases the flame shifts towards the fuel stream to a lower oxygen concentration zone. Then, upon increasing our fixed number of the oxidizer Lewis number Le(Ox) to 1.4, it is shown correspondingly in Fig. 4, that such an increase in Le(Ox) leads to a flame which is located closer to the oxidizer outer flow. Note that in Fig. 4, the increase in fuel Lewis number shifts the flame location from negative η_F values to positive ones. Thus, it causes the flame sheet to change its shape from the $-\sqrt{x}$ form into a flat one and change further to acquire the $+\sqrt{x}$ shape.

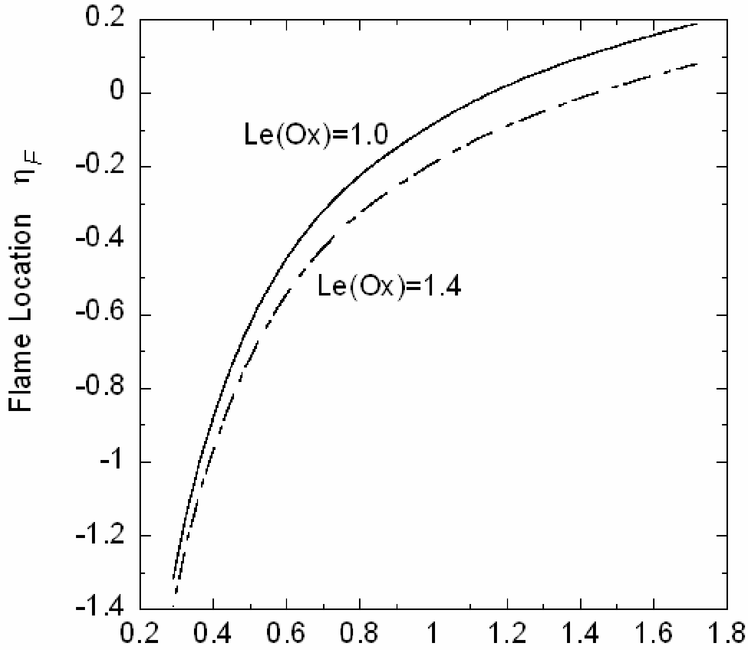


Fig. 4. Flame location as a function of fuel and oxygen Lewis numbers. Parameters used: $U_{II} / U_I = 0$, ϕ= 0.703.

The role of downstream velocity deceleration

We have examined the effect of downstream flow variation, and specifically flow deceleration, on both, the flame location in the similarity plane as well as on its shape in the physical two-dimensional plane. The constant velocity is represented here by the power α=0 in Eqs. (1) and (2), while α= −0.1, is for a downstream decelerating flow. As already mentioned, zero α value leads to a single solution for the momentum equations, while when this power is negative there is an infinite number of solutions that satisfy the boundary conditions. Thus in such a complex case, one sets a criterion for choosing a solution, and it has been found (Katoshevski et al. 1993) that only one solution admits an exponential behavior of the decay of the velocity profile as it reaches the outer flows boundary conditions. In contrast to the constant velocity case, as mentioned earlier with respect to Eqs. (6) and (7), here, a restriction is unavoidable, which is $(U_{II} / U_I)^2 = \rho_I / \rho_{II}$. Fig. 5 accounts for that restriction, unlike Fig. 1-4.

The change in the velocity profile from the constant-velocity case, α=0, to the decelerating case, α= −0.1, has an effect on the flame location in the similarity plane, as shown in Fig. 5. A downstream deceleration shifts up the flame towards the fuel stream as the extent of forced convection across the shear layer becomes weaker and the stoichiometric conditions are obtained in an upper area in the fuel stream. The figure also shows that as the equivalence ratio increases the deviation between the two lines is changing, hence, the role of the equivalence ratio is somewhat different for different behavior of flow downstream variation.

Finally, the effect of downstream deceleration on the physical shape of flame sheet in the x-y plane is described in Fig. 6. The effect is significant, even when, for simplicity, considering the same η_F value. This, in addition to an upper shift in the value of η_F described in Fig. 5 implies that a downstream velocity deceleration not only shifts the flame location towards the fuel stream but also changes the curvature of the flame.

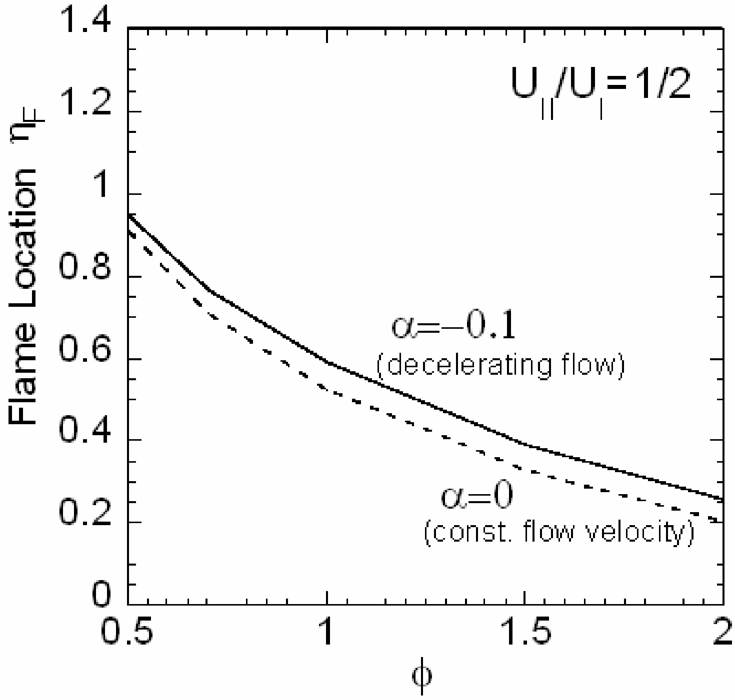


Fig. 5. Effect of flow deceleration on flame location, for *Sc*=1.

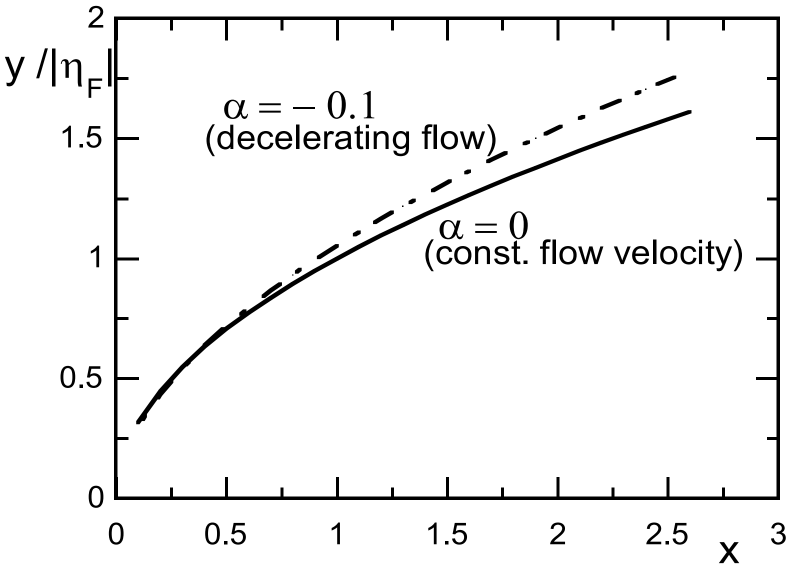


Fig. 6. Effect of flow deceleration on flame shape for the same η_F.

CLOSURE

The mathematical study presented here of a diffusion flame in shear layer flow elucidates the profound role of the velocity distribution across the shear layer. An equivalence ratio "turning point" is shown to exist in the shift of the flame location from one stream towards the other with increasing Schmidt number. Here as well, the value of the corresponding "turning point" equivalence ratio where the shift changes direction is found to be governed by the velocity distribution which is driven by the free stream velocity ratio U_{II} / U_I. In addition, this ratio governs also the sensitivity of the flame location to changes in Schmidt number. This sensitivity is reduced with increasing velocity ratio.

From another point of view, for a fixed value of the equivalence ratio, the Schmidt number is shown to play the role of a stabilizer which forces the flame to be situated closer to the interface between the two streams. This becomes more profound as the velocity ratio decreases.

The effect of downstream velocity deceleration is also addressed, in terms of flame location and shape. It is presented as a function of the equivalence ratio implying that the role of the equivalence ratio on flame location is influenced by downstream velocity variation.

The current analytical formulation could serve as a practical tool for the prediction of the trends in flame location and shape as a function of the velocity field and of various fuel and oxidizer species involved in the formation of such a diffusion flame, as well as a test case for a more detailed and numerical analysis. Also, the trends shown here, in response to changes of the various operating conditions can be used to manipulate flame characteristics and hence to affect the out-coming emissions from fires in related configurations.

LIST OF NOTATIONS

f	function of η
Le	Lewis number
m	mass fraction
M	molecular weight
P	normalized pressure
Pr	Prandtl number
Sc	Schmidt number
$\bar{T}$	normalized temperature
u	longitudinal velocity of the gas flow
x	normalized longitudinal coordinate
$\hat{x}_c$	characteristic longitudinal distance
y	normalized lateral coordinate

α	power of x in the expression for the outer velocity U
η	similarity variable
ρ	density
μ	viscosity
$\hat{\nu}$	stoichiometric ratio
ψ	stream function
ϕ	equivalence ratio, $\hat{\nu} m_{f,\infty} / m_{o,-\infty}$

Subscripts

I	denotes the fuel stream
II	denotes the oxidizer stream
f	fuel
o	oxidizer
F	indicates the flame sheet
$\pm\infty$	indicates the outer free streams

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