

## **Characteristics of Spray Grouping/ Non-Grouping Behavior**

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### **Abstract**

Travel grouping of particles/droplets in an oscillating flow is described. The tendency for non-grouping behavior is characterized by a new non-dimensional parameter designated as  $NG$ . A characteristic time for grouping is also presented. The prediction of grouping tendency and the characteristic time are incorporated in a spray evaporation model for estimating the effect of grouping on the overall vapor production of a multi-sized spray. It is shown that the relatively small droplets tend to form groups and are thus expected to undergo a decrease in their evaporation rate. Larger droplets tend not to group, thus their rate of evaporation is maximized. Therefore, such travel grouping brings the evaporation rates of different droplet size ranges closer together, implying that an optimized fuel spray injected into a combustion system is not necessarily characterized by the smallest Sauter mean diameter (SMD). It is also shown that once a grouping or a non-grouping mode of behavior has been established, evaporation has only a minor effect on this behavior during droplet lifetime. Thus, the initial operating conditions, which are incorporated in the initial value of  $NG$ , can predict the extent of grouping stability.

**Keywords:** Spray Dynamics, Sectional Method

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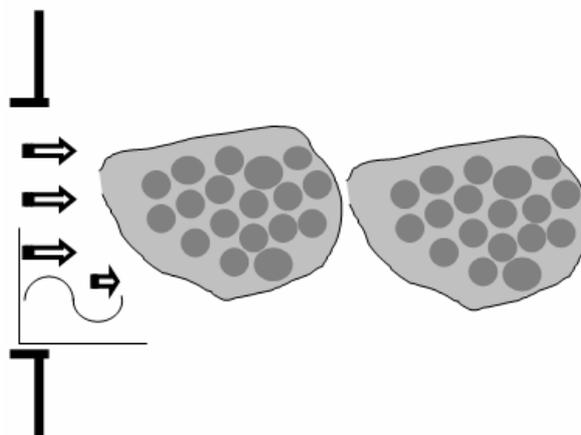
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## INTRODUCTION

The interaction of vaporizing droplets with an oscillating flow field has a variety of industrial applications. One such an application involves the vaporization of fuel droplets in a combustion chamber. The spatial spreading of the fuel vapors is a key parameter influencing the combustion characteristics; spreading of vapors is governed primarily by droplet trajectories and by the local concentration of the liquid phase (Tambour, 1985). In the current study, we examine the phenomenon that we designate as “travel grouping,” which refers to the tendency of droplets to gather into groups (see, for example, Bellan and Harstad, 1990) as a function of the operating conditions. We deal with multi-sized spray droplets which evaporate and may group according to the operating conditions. Figure 1 presents a schematic description of the general configuration in which droplets in motion in an oscillating flow field are in the process of forming groups. The gray-colored clouds represent vapors.

The effect of grouping on the evaporation rate has been the subject of a number of studies (a partial list is given in Sirignano, 1999). The present study allows us to predict when the models for evaporation rates described in those studies may be used. However, modification of the evaporation rate is addressed only in a qualitative manner.

Some aspects of the equation of motion of droplets associated with the effect of the oscillations of the host gas flow have already been addressed by Katoshevski et al. (2005). Since the current treatment is based on the previous work (Katoshevski et al., 2005), the relevant aspects are described briefly below. The innovations of the current study lie in the description of a new criterion for non-grouping tendency and in a new expression for the time until grouping is established for cases when grouping is expected. In addition, the effect of oscillation frequency and droplet size on grouping behavior is demonstrated. Finally, the mutual effect of dynamics of the droplets and their evaporation is addressed, followed by further demonstration of this phenomenon.



**Figure 1.** Schematic description of travel-grouping in an oscillating flow field.

## **DROPLET MOTION AND GROUPING/NON-GROUPING CRITERIA**

Consider the motion of a droplet in a one-dimensional periodic fluid flow, as described by eqn 1 (Katoshevski et al., 2005):

$$u(t, x) = U_a - U_b \sin(kx - \omega t) \quad (1)$$

where  $u(t, x)$  is the fluid velocity at time  $t$  at location  $x$ ,  $U_a$  is the mean flow velocity,  $U_b$  is the amplitude of the velocity oscillation,  $k$  is the wave number, and  $\omega$  is the angular velocity. Without loss of generality, we assume that  $U_a, U_b > 0$ . The flow, described by eqn 1, has a period  $L = 2\pi / k$  along the  $x$ -axis and a time period  $T = 2\pi / \omega$ . The phase velocity of the wave is  $U_w = \omega / k$ .

Normalizing the velocities by  $U_w$ , the distance by  $k^{-1}$ , and the time by  $\omega^{-1}$  leads to the following equation of droplet motion:

$$\frac{du_j}{dt} = \frac{1}{St} (u - u_j), \quad (2)$$

where  $St = \frac{1}{18} \frac{\rho_d \omega D_d^2}{\mu}$  is the Stokes number,  $\rho_d$  is the density of the liquid,  $D_d$  is the diameter of droplet, and  $\mu$  is the dynamic viscosity of the host gas.

Introducing a new variable,  $\theta = x - t$ , we may rewrite eqn 2 in a more explicit form:

$$\frac{d^2\theta}{dt^2} + \frac{1}{St} \frac{d\theta}{dt} + \frac{U_b}{St} \sin\theta = \frac{U_a - 1}{St}. \quad (3)$$

Eqn 3 can be simplified to:

$$\theta'' + \alpha\theta' + \sin\theta = \beta, \quad (4)$$

where the primes denote derivatives with respect to  $\tau$ , defined as  $\tau = t \sqrt{U_b / St}$ ,

$$\alpha = 1 / \sqrt{U_b St} \quad (5)$$

And

$$\beta = (U_a - 1)/U_b. \quad (6)$$

Eqn 2 is valid for low values of the Reynolds ( $Re$ ) numbers, but for accuracy of the calculations when  $Re > 1$  we have included a drag coefficient  $C_D$  as a function of the Reynolds number (Seinfeld and Pandis, 1998).

Examination of eqn 4 shows that for  $\theta$  we can easily find a solution without accounting for the initial condition. In that solution,  $\theta$  has a constant value, determined by the relationship,  $\sin\theta = \beta$ , i.e.:

$$\theta = \sin^{-1} \beta \quad (7)$$

A constant value of  $\theta$  means a constant velocity equal to the wave propagation velocity  $U_w$  (i.e.,  $\omega/k$ ). Indeed, the analysis of Katoshevski et al. (2005) showed that when grouping occurs, the group travels at constant velocity equal to  $U_w$ . In addition, the above-mentioned relationship,  $\sin\theta = \beta$ , also implies that a criterion for grouping would be  $\beta \leq 1$ . This approach was not covered by the previous study of Katoshevski et al. (2005), although a numerical analysis of that work revealed that when  $\beta = 1$ , the grouping becomes unstable. Hence, the criterion that is necessary for grouping is  $\beta < 1$ . However, this criterion it is not sufficient to ensure grouping, since within this range of  $\beta$ , grouping behavior also depends on  $\alpha$ . For each value of  $\beta$  in that range,  $\alpha$  must be above a certain value to ensure grouping (Katoshevski et al., 2005).

A non-grouping situation is desired in most liquid-spray systems so as to maximize evaporation. Hence, from that point of view, it is important to obtain a new criterion based on  $\alpha$ - $\beta$  relationship, characterizing the tendency to a non-grouping behavior. Such a criterion is presented here in terms of a new, non-dimensional parameter, denoted as  $NG$  (which stands for non-grouping, and is the ratio between  $\beta$  and  $\alpha$ ):

$$NG = \frac{U_a - 1}{\sqrt{U_b}} \sqrt{St} \gg 1 \quad (8)$$

$NG$  must be  $\gg 1$  to ensure a non-grouping mode of behavior. The above range for  $NG$  coincides with our numerical calculations. According to our numerical investigation, grouping becomes unstable when  $NG$  is of the order of  $\geq 1$ .

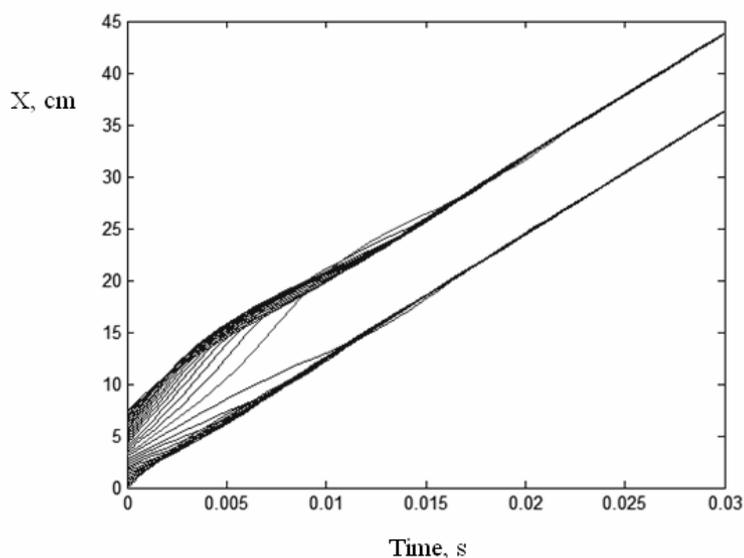
The new parameter  $NG$  is a function, among others, of the oscillation frequency and the droplet size. An increase in the frequency or the droplet size will result in an increase of  $NG$ , thus leading to a non-grouping tendency. The same trend will occur when the density of the liquid is increased.

The above criterion can be used for correcting the predicted evaporation rates according to grouping/non-grouping situations. This will be addressed in the next section, but first let us examine some features of grouping behavior in terms of the new criterion, as shown in Figures 2-5.

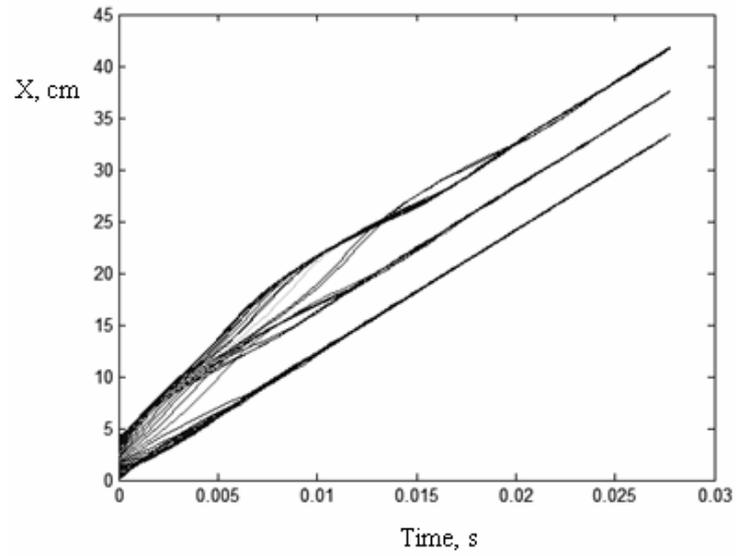
Figure 2 presents a clear case of grouping, in which droplets 30  $\mu\text{m}$  in diameter that are dispersed along a distance of one cycle ( $L$  in length) form two travel groups. We term such grouping behavior as "stable-grouping." For that case, the value of  $NG$  is 0.78. When  $\omega$  (which is linear with frequency) is increased from 1000 to 1800 rad/s,  $NG$  becomes 1.04, and one of the two groups splits up into two; i.e., there is a secondary breakup of the overall group, as shown in Figure 3. This behavior is termed "unstable grouping."

A further increase of  $\omega$  to 2000 rad/s leads to an increase of  $NG$  to 1.11 and causes a second stage of group breakup (Fig. 4). If we now increase the droplet size to 70  $\mu\text{m}$  and  $\omega$  to 6000 rad/s, we obtain a clear case of non-grouping case, as presented in Figure 5. In that case,  $NG$  is well over unity (equals 4.45), as in the criterion of eqn 8.

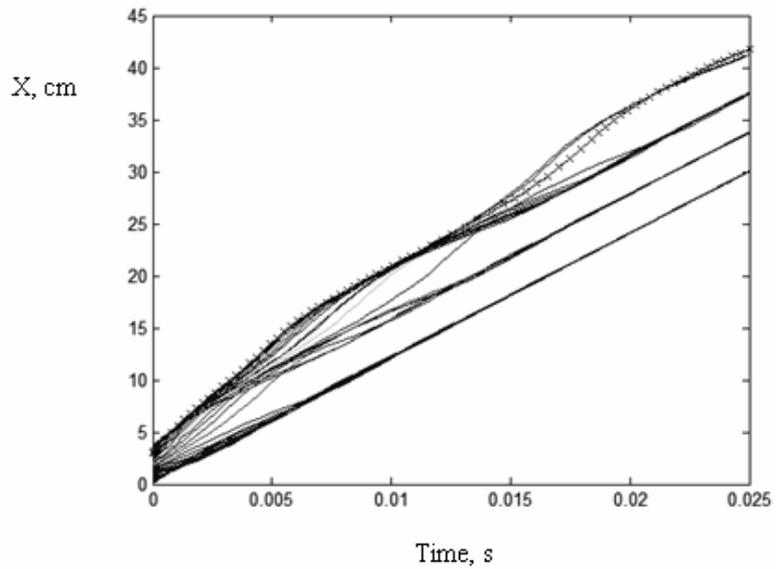
Thus, Figures 2-5 demonstrate how the parameter  $NG$  may be used as an indicator of grouping behavior, from stable grouping, when  $NG$  is of the order of less than 1, through non-stable grouping when  $NG$  is of the order of 1, to non-grouping, when  $NG$  is greater than 1.



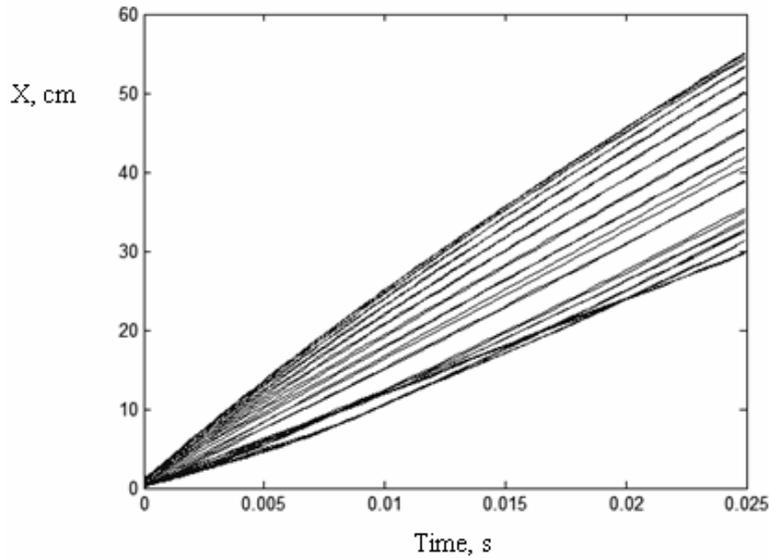
**Figure 2.** Trajectories of droplets in the grouping mode for droplets of  $D_d = 30 \mu\text{m}$  and  $\omega = 1000 \text{ rad/s}$ .



**Figure 3.** Non-grouping mode for droplets of  $D_d = 30\mu\text{m}$  and  $\omega = 1800 \text{ rad/s}$ .



**Figure 4.** Non-grouping mode for droplets of  $D_d = 30\mu\text{m}$  and  $\omega = 2000 \text{ rad/s}$ .



**Figure 5.** Non-grouping mode, for droplets of  $D_d = 70\mu\text{m}$  and  $\omega = 6000 \text{ rad/s}$ .

To account for the grouping effect on evaporation, it is important to obtain an expression for the approximate time during which grouping is established. In light of the criteria for stable grouping, i.e.,  $\beta < 1$  and  $\alpha \gg \beta$ , we examine the various orders of magnitude of the terms in eqn 4 when  $\alpha \gg 1$ . It is easy to show that, as mentioned earlier, without accounting for the initial condition, a solution to the full equation takes the form of a constant value of  $\theta$ . This solution is also in agreement with the leading term of  $\alpha\theta'$  being equal to zero. The next order of the equation involves a linearization of the sine term by a small angle approximation, leading to  $\theta'' + \theta = \beta$ . This, in turn, implies that the correction terms involve  $\sin(\tau)$ ,  $\cos(\tau)$  and  $\beta$ . When grouping is achieved, the group velocity becomes equal to the wave velocity  $U_G = \omega/k$ , and  $\theta$  reaches a constant value. We thus sought a value of  $\tau$  that would reduce the effect of the correction term, on the one hand, and would agree with the numerical analysis, on the other hand. This value, found to be  $\tau = 2\pi$ , may be applied for estimating the non-dimensional time required for grouping to be established:

$$t_G = 2\pi \sqrt{\frac{St}{U_b}} \tag{9}$$

For example, with respect to the parameters of Figure 2, the non-dimensional time corresponds to a dimensional time of 0.011 s, a value that is in good agreement with the trajectories shown in Figure 2.

## TRAVEL GROUPING AND EVAPORATION COUPLING

We now show how to incorporate the effect of grouping on evaporation, and vice versa; i.e., how evaporation affects grouping. For our qualitative description, we choose to employ the Lagrangian sectional method (Tambour, 1985; Greenberg, 1989); i.e., to divide the overall size range into size-groups (sections) and thereby to obtain a set of sectional-differential equations in the form:

$$\frac{dQ_j}{dt} = -C_j Q_j + B_{j,j+1} Q_{j+1} \quad ; \quad j = 1, 2, \dots, N_s \quad (10)$$

where  $N_s$  denotes the total number of size-sections, and  $C_j$  and  $B_{j,j+1}$  are the sectional evaporation coefficients. The coefficient  $B_{j,j+1}$  accounts for droplets from section  $(j+1)$ , which are added to section  $j$  during evaporation, whereas  $C_j$  accounts for evaporation of droplets within section  $j$  and for droplets that “move” from section  $j$  to section  $(j-1)$ . These sectional evaporation coefficients are, in general, a function of the size range of the section, the temperature, and the characteristics of the liquid. They may also incorporate the effect of the difference between velocity of the droplet and that of the host gas by a Ranz-Marshall type of correlation (Ranz and Marshall, 1952), as was done, for example, by Katoshevski and Tambour (2000). The effect of droplet coalescence can also be taken into account by adding the appropriate terms (Tambour, 1985), but this effect is not within the scope of the current work.

As mentioned above, we used a Lagrangian type of approach in our approximations, and for the estimation of overall production of vapors we may write:

$$\frac{dm}{dt} = S_v \quad (11)$$

where,

$$S_v = - \sum_{j=1}^{N_s} (-C_j Q_j + B_{j,j+1} Q_{j+1}) \quad (12)$$

When dealing with the production of vapors with specific, distinct ranges of droplet sizes, we sum up the contributions of the various groups to the overall vapor production. In addition, we estimate the overall vapor production in the grouping mode by dividing the time frame into two periods: 1) up to  $t_G$ , and 2) after  $t_G$ . In the second period, the evaporation rate is expected to be lower than that of isolated

droplets. This effect may be taken into account by introducing a correction factor  $E_j$  into the sectional evaporation coefficients  $C_j$  and  $B_{j,j+1}$ :

$$E_j = \begin{cases} < 1, \beta < 1, NG < 1, t > t_G \\ 1 \end{cases} \quad (13)$$

When  $E_j < 1$ , the correction term suggested by Chiu et al. (1983) for evaporation and combustion of non-isolated droplets may be used.  $E_j$  depends mainly on the Lewis number, ( $Le$ ), the total number of droplets in the group  $N$ , the averaged droplet diameter  $D_d$ , and the spacing between the centers of the droplets  $s$ .  $E_j$  may be approximated by the following parameter:

$$G \sim Le N^{2/3} D_d / s. \quad (14)$$

Thus, we may further refine the sectional evaporation coefficients, which are a function of the sectional size range, the properties of the liquid, and the temperature, by applying the above  $E_j$  function. Thus, we may write:

$$C_j, B_{j,j+1} \sim E_j(G, \beta, NG, t_G). \quad (15)$$

Hence, the overall vapor production from a multi-sized spray undergoing grouping may be estimated as:

$$m(t) = \int_0^{t_G} \left[ -\sum_{j=1}^{N_s} (-\bar{C}_j Q_j + \bar{B}_{j,j+1} Q_{j+1}) \right] dt + \int_{t_G}^t \left[ -\sum_{j=1}^{N_s} (-E_j \bar{C}_j Q_j + E_j \bar{B}_{j,j+1} Q_{j+1}) \right] dt \quad (16)$$

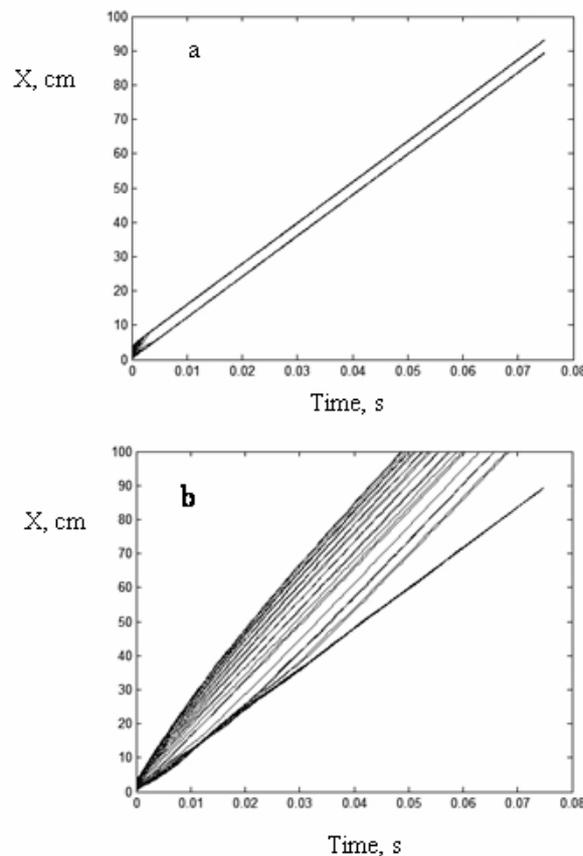
where the sectional evaporation coefficients  $\bar{C}_j, \bar{B}_{j,j+1}$  refer to the non-grouping case.

The same approach may be applied for the analysis of droplet coalescence (Tambour, 1985). Since grouping enhances the sectional rates of coalescence, these rates should be adjusted at times greater than  $t_G$ .

## DEMONSTRATION AND CONCLUSIONS

To demonstrate the relationships developed above, let us consider the case of a spray that initially has a bimodal size distribution; i.e., a spray comprised of two distinct droplet sizes, say, 10  $\mu\text{m}$  and 70  $\mu\text{m}$  in diameter. The trajectories of these droplets, without evaporation, are presented in Figures 6a and 6b. For the same flow field, at  $t = t_G$  the small droplets ( $D_d = 10 \mu\text{m}$ ) form groups (Fig. 6a), whereas the large droplets ( $D_d = 70 \mu\text{m}$ ) do not (Fig. 6b).

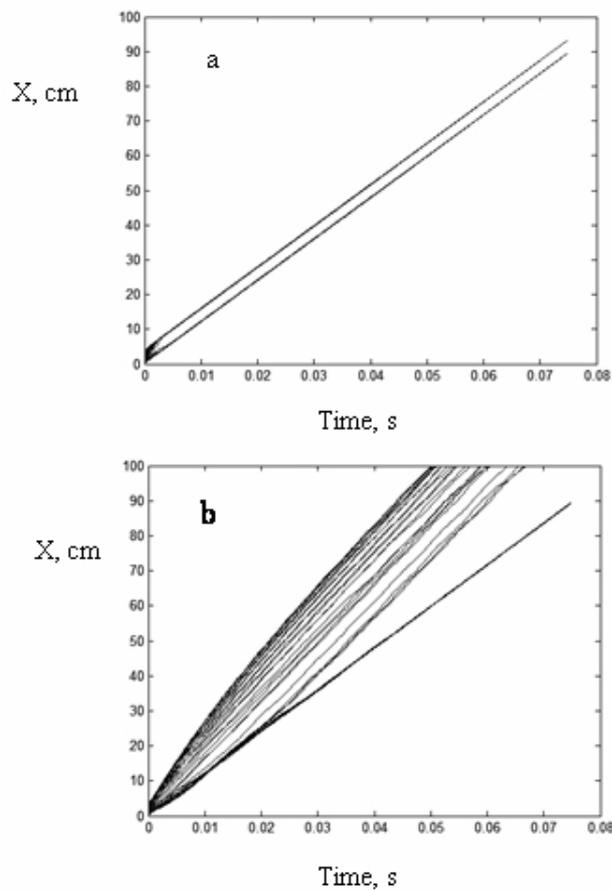
Next, we add the effect of droplet size reduction by the  $D^2$ -law of evaporation. For a qualitative demonstration that requires us to be consistent with the time scale of the former figures, we need to arrange for the square of the droplet diameter to be reduced dramatically over a time of 0.08 s. Thus, in Figure 7a, the droplet diameter is reduced from 10  $\mu\text{m}$  to 0.5  $\mu\text{m}$ . A comparison of Figure 7a with Figure 6a shows that the reduction in droplet size has a very little effect, if any, on the grouping behavior of these small droplets. This finding coincides with the above-mentioned conclusion that smaller droplets have higher tendency to group. Thus, after grouping has occurred, a reduction in size will not break the group but may strengthen grouping tendency.



**Figure 6.** Trajectories of non-evaporating droplets: (a)  $D_d = 10 \mu\text{m}$ , and (b)  $D_d = 70 \mu\text{m}$ ;  $\omega = 2000 \text{ rad/s}$ .

The size reduction of the larger droplets (Fig. 7b), from a diameter of 70  $\mu\text{m}$  to a diameter of 6  $\mu\text{m}$  over 0.08 s, requires a higher evaporation rate than that of the smaller droplets (Fig. 7a), in practical this would mean that we require a different liquid or a different temperature. Although the size of these relatively large droplets is reduced considerably with time as they evaporate, our calculations show that the tendency of these droplets to group is low. Thus, for large droplets, which do not initially form a travel group, evaporation and a simultaneous reduction in size are not expected to lead to stable grouping of the spray as a whole during the droplet lifetime. Such a spray would be characterized by an initial  $NG$  value that is larger than unity.

Let us now consider the effect of a “slow-down” in evaporation as grouping occurs. This may be done by employing the function  $Ej$  of eqn 13. Grouping is relevant to the small droplets, as has already been shown in Figures 6 and 7. Thus, we do not need to apply the function  $Ej$  for the calculation of the large droplets of Figure 7b. As has already been concluded from Figures 6 and 7, once grouping has been established, the rate of evaporation does not change the grouping behavior of the small droplets during their lifetime. Here, too, the slow-down in evaporation does not change the trajectories of Figure 7a.



**Figure 7.** Trajectories of evaporating droplets: (a)  $D_d = 10 \mu\text{m}$ , and (b)  $D_d = 70 \mu\text{m}$ ;  $\omega = 2000 \text{ rad/s}$ .

What remains for us to estimate is the change in vapor production or the change in evaporation rate of the group of small droplets compared with the same number of isolated droplets. Labowsky (1980) has shown that the  $D^2$ -law essentially describes the evaporation rate of non-isolated droplets, but with a decreasing factor, represented here by the function  $E_j$ . That study has shown that, depending on spacing between droplets and the operating conditions, the reduction in the evaporation rate may be substantially larger than 50%. Since the aim of our current simple mathematical modeling is to represent a qualitative picture, we may use such an estimation of the reduction in the evaporation rate for purposes of demonstration. By applying the study of Tambour (1985) on the sectional approach (eqns 9 and 16), we estimate that for isolated droplets the averaged sectional evaporation coefficient  $C_j$  of a droplet with initial size of 10  $\mu\text{m}$  is three times the corresponding value for a 70- $\mu\text{m}$  droplet. Thus, as a result of grouping the evaporation rates of these two classes of sizes may approach similar values. The amounts of vapors produced can be evaluated by eqns 9 and 16, and as a first-order approximation and for  $t > t_G$ , we may write:

$$m(t) = Q_{70}(0) \left[ 1 - \text{Exp}(-\tilde{C}_{70} t) \right] + Q_{10}(0) \left( 1 - \text{Exp}(-\tilde{C}_{10} t_G) \right) \left[ 2 - \text{Exp}(-E\tilde{C}_{10} (t - t_G)) \right] \quad (17)$$

where the terms with tildes ( $\sim$ ) represent an averaged value of vaporization rate. As a first approximation, the values of the  $B_j$  (the other sectional evaporation coefficients) may be neglected, since they are smaller than the values of  $C_j$  (Tambour, 1985) and since they are initially zero for the two distinct size classes considered here. The calibration of the coefficients in eqn 17 can be carried out experimentally.

In conclusion, drops of smaller sizes tend to form groups and are thus expected to undergo a decrease in evaporation rate, while larger droplets do not tend to group, thus maximizing the rate of vaporization. These tendencies may serve to bring the evaporation rates of different size ranges closer together. Thus, when oscillations are evident in the flow field, the optimal spray-injector is not necessarily the one producing the smallest droplet Sauter mean diameter (SMD).

Both the new criterion for non-grouping introduced here and the characteristic time for grouping for the cases when grouping is expected can serve as tools to be used in the design of spray injectors and of other devices in which multi-sized particles or droplets are subjected to an oscillatory flow field.

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